BOOLEAN ALGEBRA

Basic Laws of Boolean Algebra

Commutative law:

A + B = B + A
B + A = A + B

Associative law:

A + (B + C) = (A + B) + C
A. (B.C) = (A.B).C

Distributive law

A. (B + C) = A.B + A.C

Other laws of Boolean algebra:

1. A + 0 = A

2. A + 1 = 1

3. A + A = A

4. $A + \bar{A} = 1$

5. A .0 = 0

6. $A . 1 = A$

7. $A . A = A$

8. $A . \bar{A} = 0$

9. $\bar{\bar{A}} = A$

10. $A + A.B = A$

11. $A.(A + B) = A$

12. $(A + B).(A+C) = A + B.C$

13. $A + \bar{A}.B = A + B$

14. $A.(\bar{A} + B) = A.B$

15. $(A + B).(\bar{A} + C) = A.C + \bar{A}.B$

16. $(A + C).(\bar{A} + B) = A.B + \bar{A}.C$

De Morgan’s Theorems:

I Theorem statement:

The complement of a sum is equal to the product of the complements.

$\overline{A + B} = \bar{A} . \bar{B}$

II Theorem Statement:

The complement of a product is equal to the sum of the complements.

$\overline{A . B} = \bar{A} + \bar{B}$

Proof of first theorem:

To prove $\overline{A + B} = \bar{A} . \bar{B}$

Case 1: A=0, B=0

L.H.S => $\overline{A + B} = \overline{0 + 0} = \bar{0} = 1$

R.H.S => $\bar{A} . \bar{B} = \bar{0} . \bar{0} = 1 . 1 = 1$

Case 2: A=0, B=1

L.H.S => $\overline{A + B} = \overline{0 + 1} = \overline{1} = 0$

R.H.S => $\overline{A} . \overline{B} = \overline{0} . \overline{1} = 1 .0 = 0$

Case 3: A=1, B=0

L.H.S => $\overline{A + B} = \overline{1 + 0} = \overline{1} = 0$

R.H.S => $\overline{A} . \overline{B} = \overline{1} . \overline{0} = 0 .1 = 0$

Case 4: A=1, B=1

L.H.S => $\overline{A + B} = \overline{1 + 1} = \overline{1} = 0$

R.H.S => $\overline{A} . \overline{B} = \overline{1} . \overline{1} = 0 .0 = 0$

Truth table

A	B	$\overline{A + B}$	$\overline{A} . \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Proof of second theorem:

To prove $\overline{A . B} = \overline{A} + \overline{B}$

Case 1: A=0, B=0

L.H.S => $\overline{A . B} = \overline{0 . 0} = \overline{0} = 1$

R.H.S => $\overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 + 1 = 1$

Case 2: A=0, B=1

L.H.S => $\overline{A . B} = \overline{0 . 1} = \overline{0} = 1$

R.H.S => A + B = 0 + 1 = 1 + 0 = 1

Case 3: A=1, B=0

L.H.S => $\overline{A . B} = \overline{1 . 0} = \overline{0} = 1$

R.H.S => $\overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Case 4: A=1, B=1

L.H.S => $\overline{A . B} = \overline{1 . 1} = \overline{1} = 0$

R.H.S => $\overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$

Truth table

A	B	$\overline{A . B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

CHECK YOUR PROGRESS 2

1. An inverter performs an operation known as
(a) Complementation (b) assertion
(c) Inversion (d) both answers (a) and (c)

2.The output of gate is LOW when at least one of its inputs is HIGH. It is true for
(a) AND (b) NAND (c) OR (d) NOR

3.The output of gate is HIGH when at least one of its inputs is LOW. It is true for
(a) AND (b) OR (c) NAND (d) NOR

4.The output of a gate is HIGH if and only if all its inputs are HIGH. It is true for
(a)XOR (b) AND (c) OR (d) NAND

5. The output of a gate is LOW if and only if all its inputs are HIGH. It is true for
(a)AND (b) XNOR (c) NOR (d) NAND

6.Which of the following gates cannot be used as an inverter?

(a)NAND (b) AND (c) NOR (d) None of the above

7.The complement of a variable is always
(a) 0 (b) 1 (c) equal to the variable (d) the inverse of the variable

8.Which one of the following is not a valid rule of Boolean algebra?
(a) A + 1 = 1 (b) $A = \bar{A}$ (c) A.A = A (d) A + 0 = A

9. Which of the following rules states that if one input of an AND gate is always 1 , the output is equal to the other input ?
(a) A + 1 = 1 (b) A + A = A (c) A.A = A (d) A . 1 = A

SUMMARY

- A binary number is a weighted number in which the weight of each whole number digit is a positive power of 2 and the weight of each fractional digit is a negative power of 2.
- The 1’s complement of a binary number is derived by changing 1s to 0s and 0s to 1s
- The 2’s complement of a binary number can be derived by adding 1 to the 1’s complement.
- The octal number system consists of eight digits, 0 through 7.
- The hexadecimal number system consists of 16 digits and characters, 0 through 9 followed by A through F.
- The ASCII is a 7-bit alphanumeric code that is widely used in computer systems for input/output of information.
- The output of an inverter is the complement of its input
- The output of an AND gate is high only if all the inputs are high
- The output of an OR gate is high if any of the inputs is high
- The output of an NOR gate is low if any of the inputs is high
- The output of an NAND gate is low only if all the inputs are high
- The output of an exclusive-OR gate is high when the inputs are not the same