* Complex Numbers. *

* Define complex Number :-

A Number which can be written in the form of $x+iy$, where x and y are Real Numbers is called a Complex Number.

e.g :- $2+3i$, $4+5i$

* Real and Imaginary Part of a Complex Number

Let $z = x+iy$ be a complex Number.

Then 'x' is called the Real ~~Number~~ Part of z. The Real part of z is denoted by $Re(z)$.

And 'y' is called the Imaginary part of z. The Imaginary part of z is denoted by $Im(z)$.

e.g If $z = 2+3i$ then

Real part of $z = Re(z) = 2$

and Imaginary part of $z = Im(z) = 3$

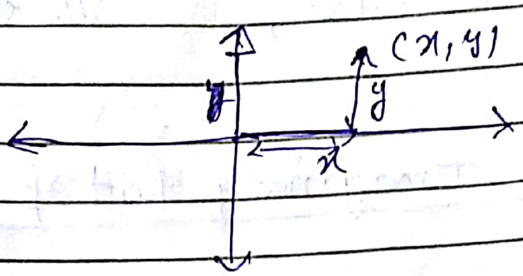
* जिसके साथ i (iota) नहीं होता तो Real Part Hota है और जिसके साथ i (iota) होता है तो Imaginary Part Hota है। *

18

DATE: / 20
PAGE No

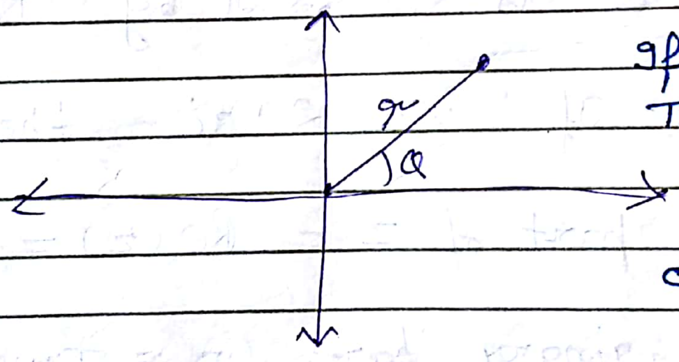
* Cartesian Form of a complex Number.

Let $z = x+iy$ be a complex Number. It is Represented by a point (x, y) in a plane, is Called Cartesian Form.



* Polar Form of a Complex Number

Let $z = r\cos\theta + ir\sin\theta$
$= r(\cos\theta + i\sin\theta)$
is called the polar form of a complex Number. and it is denoted by $r\angle\theta$.



If $z = x+iy$
Then
$$r = \sqrt{x^2+y^2}$$
$$\theta = \tan^{-1}\left|\frac{Im(z)}{Re(z)}\right|$$
$$= \tan^{-1}\left|\frac{y}{x}\right|$$

* Define Modulus of a Complex No:-

The Modulus of a complex No. $z = x+iy$ is denoted by $|z|$ and is defined by

$$|z| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(\text{Real Part})^2 + (\text{Imaginary Part})^2}$$

* **Argument or Amplitude of a Complex No :-**

Let $z = x+iy$ be any Complex Number. Then angle $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ is called Argument or Amplitude of a complex No. and is denoted by $\arg(z)$ or $\text{amp}(z)$.

Note :- The value of θ which satisfy the inequality $-\pi < \theta < \pi$ is known as Principal argument of z.

* **Addition of a Complex No :-**

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex No.

Then $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$

$$= (x_1 + x_2) + i(y_1 + y_2)$$

* **Define Subtraction of a complex No :**

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex No. Then $z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2)$

$$= (x_1 - x_2) + i(y_1 - y_2).$$

* Define Multiplication of Two Complex No.

Sol:- Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex No. Then Their Multiplication is denoted by

$$z_1 z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2)$$

$$= x_1 x_2 + x_1 \cdot iy_2 + iy_1 x_2 + i^2 y_1 y_2$$

$$= x_1 x_2 + x_1 y_2 i + x_2 y_1 i - y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

* Define division of Two complex No.

Sol:- Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex No. Then Their division is denoted by

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

* Define conjugate of a complex No.

Let $z = x + iy$ be a complex No. Then conjugate of z is denoted by $\bar{z}$ and is equal to $\bar{z} = x - iy$.

(जब भी हम conjugate लेते है तब i से पहले जो निशान होता है उसको बदलना होता है। मतलब अगर i से पहले '+' है तो उसे '−' कर दो अगर i से पहले '−' है तो उसे '+' कर दो)

Q1:- Evaluate

(i) i^{135}

Solution:-

$$i^{135} = (i^4)^{33} \cdot i^3$$

$$= (1)^{33}(-i) = 1 \times (-i)$$

$$= -i \text{ Ans}$$

Note:- $i = \sqrt{-1}$

$\therefore \quad i^2 = (\sqrt{-1})^2 = -1$

$\therefore \quad i^3 = i^2 \cdot i = (-1)(i) = -i$

$\therefore \quad i^4 = (i^2)^2 = (-1)^2 = 1$

$\therefore \quad i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

22

DATE: / 20
PAGE No

(ii) i^{-999}

$= \frac{1}{i^{999}} = \frac{1}{(i^4)^{249} \cdot i^3}$

$= \frac{1}{(1)^{249}(-i)} = \frac{1}{1 \times (-i)}$

$= \frac{1}{-i} = -\frac{1}{i}$

$= -\frac{1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{+i}{+1}$

$= i$ Ans.

(iii) i^{365}

$= (i^4)^{91} \cdot i^1$

$= (1)^{99}(i) = (1)(i)$

$= i$ Ans.

Q2 Show that

(i) $1 + i^{10} + i^{20} + i^{30}$ is a real no.

Solution: $1 + (i^4)^2 i^2 + (i^4)^5 + (i^4)^7 \cdot i^2$

$= 1 + (1)^2(-1) + (1)^5 + (1)^7(-1)$

$= 1 - 1 + 1 - 1 = 0$

(ii) $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in N$
For all

Solution:- Taking common i^n from L.H.S

$= i^n(1 + i + i^2 + i^3)$

$= i^n(\cancel{1} + \cancel{i} - \cancel{1} - \cancel{i})$

$= i^n(0)$

$= 0 = R.H.S.$

Q3:- Express each one of the following in the standard form $x+iy$.

(i) $\dfrac{5+4i}{4+5i}$

Note:- Standard form में बदलने के लिए statement में नीचे वाली Term से i हटाना है और उसको हटाने के लिए हमें Rationalize करना पड़ता है। Rationalize के लिए नीचे वाली Term का निशान बदल कर ऊपर और नीचे गुणा करनी होती है।

Solution:- $\dfrac{5+4i}{4+5i} \times \dfrac{4-5i}{4-5i}$

$= \dfrac{20 - 25i + 16i - 20i^2}{(4)^2 - (5i)^2}$

$$= \frac{20 - 9i - 20(-1)}{16 - 25i^2}$$

$$= \frac{20 - 9i + 20}{16 - 25(-1)} = \frac{40 - 9i}{16 + 25}$$

$$= \frac{40 - 9i}{41}$$

$$= \frac{40}{41} - \frac{9i}{41} \quad \text{Ans.}$$

Q4 Express in the standard form $x + iy$

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Solution

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

$$= \frac{6 + 9i - 4i - 6i^2}{2 - i + 4i - 2i^2}$$

$$= \frac{6 + 5i - 6(-1)}{2 + 3i - 2(-1)}$$

$$= \frac{6 + 5i + 6}{2 + 3i + 2}$$

$$= \frac{12 + 5i}{4 + 3i}$$

Now we change it in standard form.

DATE: / 20
PAGE No

$$= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48-36i+20i-15i^2}{16-\cancel{12i}+\cancel{12i}-9i^2}$$

$$= \frac{48-16i-15(-1)}{16-9(-1)}$$

$$= \frac{48-16i+15}{16+9}$$

$$= \frac{63-16i}{25}$$

$$= \frac{63}{25} - \frac{16}{25}i \quad \text{Ans}$$

Q5 Prove that the following complex No. is purely Real: $\left(\frac{3+2i}{2-3i}\right) + \left(\frac{3-2i}{2+3i}\right)$

Soln:- $\frac{3+2i}{2-3i} + \frac{3-2i}{2+3i}$

$$= \frac{(3+2i)(2+3i) + (3-2i)(2-3i)}{(2-3i)(2+3i)} \quad [\text{Taking L.C.M}]$$

$$= \frac{6+9i+4i+6i^2+6-9i-4i+6i^2}{(2)^2-(3i)^2}$$

$$= \frac{\cancel{6}+\cancel{13i}-\cancel{6}+\cancel{6}-\cancel{9i}-\cancel{4i}-\cancel{6}}{4-9i^2}$$

$$= \frac{0}{4-9(-1)} = \frac{0}{13} = 0$$

Which is purely Real.

Q 6:- Find the Real values of x and y if

$$(1-i)x + (1+i)y = 1-3i$$

Soln:- $(1-i)x + (1+i)y = 1-3i$

$\therefore \quad x - ix + y + iy = 1-3i$

$\Rightarrow \quad (x+y) + i(-x+y) = 1-3i$

Comparing Real and Img. parts.

$$x+y = 1 \quad \text{--- (1)}$$
$$-x+y = -3 \quad \text{--- (2)}$$

Solving (1) and (2)

$$x+y=1$$
$$-x+y=-3$$

Adding

$$2y = -2$$

$$y = -\frac{2}{2} = -1$$

$\therefore \boxed{y=-1}$ Ans

Putting value of y in (1)

$$x-1=1$$
$$x=1+1$$
$$\boxed{x=2}$$ Ans.

Q7:- Find the Real values of x and y if $2+(x+iy)=5-i$

Soln:- $2+(x+iy)=5-i$

$\Rightarrow 2+x+iy=5-i$

$\Rightarrow (2+x)+iy=5-i$

Comparing Real and Img. Parts.

$2+x=5$ — ①

$y=-1$ — ②

from ① $2+x=5$

$x=5-2$

$\boxed{x=3}$ Ans

from ②

$\boxed{y=-1}$ Ans.

Q8:- Find the Real values of x and y if

$(1+i)(x+iy)=2-5i$

Soln:-

$(1+i)(x+iy)=2-5i$

$\Rightarrow x+iy+ix+i^2y=2-5i$

$\Rightarrow x+iy+ix-y=2-5i$

$\Rightarrow x-y+i(x+y)=2-5i$

Comparing Real and Img. parts.

DATE / 20
PAGE No

$x-y=2 \quad -①$
$x+y=-5 \quad -②$

Solving ① and ②

$x-y=2$
$x+y=-5$

Adding
$2x=-3$

$\boxed{x=-\frac{3}{2}}$

Putting value of x in ①

$-\frac{3}{2}-y=2$

$-y=\frac{2}{1}+\frac{3}{2}$

$-y=\frac{4+3}{2}$

$-y=\frac{7}{2}$

$\therefore \boxed{y=-\frac{7}{2}}$

Q9:- ~~Chap~~ Find the Real values of x and y if
$2x+iy=4+5i$

Solution:- $2x+iy=4+5i$
Comparing Real and Img Parts

$2x=4$ $x=\frac{4}{2}=2$ $x=2$ Ans	$y=5$ Ans

Scanned by CamScanner

Q10:- Find the Modulus and conjugate of $3-2i$

Soln:- Let $z=3-2i$

$\therefore \quad |z| = \sqrt{(Re(z))^2 + (Im(z))^2}$

$= \sqrt{9+(-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$ Ans

Conjugate of $z = \bar{z} = \overline{3-2i}$

$= 3+2i$

$\therefore \bar{z} = 3+2i$ Ans.

Q11:- Find the Modulus and conjugate of $4+i$

Soln:- Let $z = 4+i$

$\therefore \quad |z| = \sqrt{(4)^2+(1)^2}$

$= \sqrt{16+1} = \sqrt{17}$ Ans

And $\bar{z} = \overline{4+i}$

$= 4-i$ Ans

Q12 Find Modulus and conjugate of $1-i$

Soln:- Let $z = 1-i$

$\therefore |z| = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$ Ans.

$\bar{z} = \overline{1-i}$

$\bar{z} = 1+i$ Ans

Q13 Find Modulus and conjugate of $1+4i$

Soln:- Let $z = 1+4i$

$|z| = \sqrt{(1)^2 + (4)^2}$

$= \sqrt{1+16} = \sqrt{17}$ Ans

$\bar{z} = \overline{1+4i}$

$= 1-4i$ Ans.

Q14:- Find Modulus and conjugate of $4i^3 + 3i^2 + 5i$

Soln:- Let $z = 4i^3 + 3i^2 + 5i$

$= 4(-i) + 3(-1) + 5i$

$= -4i - 3 + 5i$

$z = i-3$

$z = -3+i$

$\therefore \quad |z| = \sqrt{(-3)^2+(1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ Ans.

$\bar{z} = \overline{-3+i}$

$= -3-i$ Ans.

Q14:- Find Modulus and conjugate of $\dfrac{3+2i}{4-5i}$

Soln:- Let $z = \dfrac{3+2i}{4-5i}$

$z = \dfrac{3+2i}{4-5i} \times \dfrac{4+5i}{4+5i}$

$= \dfrac{12+15i+8i+10i^2}{(4)^2-(5i)^2}$

$= \dfrac{12+15i+8i-10}{16-25i^2}$

$= \dfrac{2+23i}{16-25(-1)} = \dfrac{2+23i}{16+25}$

$z = \dfrac{2+23i}{41}$

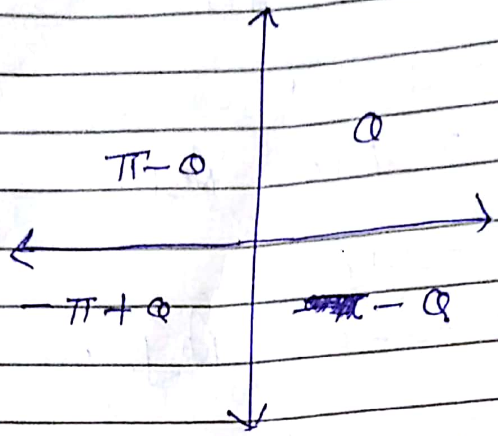
$|z| = \sqrt{\left(\dfrac{2}{41}\right)^2+\left(\dfrac{23}{41}\right)^2}$

$= \sqrt{\dfrac{4}{1681}+\dfrac{529}{1681}} = \sqrt{\dfrac{4+529}{1681}}$

$= \sqrt{\dfrac{533}{1681}} = \dfrac{\sqrt{533}}{41}$ Ans.

$\bar{z} = \overline{\dfrac{2+23i}{41}} = \dfrac{2-23i}{41}$ Ans.

* Argument $\theta = \tan^{-1}\left|\frac{y}{x}\right|$



Q15 Find Modulus and Argument of $1+i\sqrt{3}$

Soln:- Let $z = 1+i\sqrt{3}$

$$|z| = \sqrt{(1)^2+(\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$= \sqrt{4} = 2$$

$\therefore |z| = 2$ Ans.

Argument of $z = \theta = \tan^{-1}\left|\frac{y}{x}\right|$

$$\therefore \theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$$

$$= \tan^{-1}(\sqrt{3})$$

$$= \tan^{-1}\tan\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

Here $z = 1+i\sqrt{3}$ lies in I^{st} Quadrant.

$\therefore \quad \theta = \frac{\pi}{3}$ Ans.

Q16:- Find the modulus and Argument of $-2+2i\sqrt{3}$

Solution:- Let $z = -2+2i\sqrt{3}$

$\therefore \quad |z| = \sqrt{(-2)^2+(2\sqrt{3})^2}$

$= \sqrt{4+12}$

$= \sqrt{16} = 4$

Argument $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\cancel{2}\sqrt{3}}{-\cancel{2}}\right|$

$= \tan^{-1}|-\sqrt{3}|$

$= \tan^{-1}(\sqrt{3})$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

Here $x = -ve$ and $y = +ve$

$\therefore$ IInd Quadrant.

$\therefore$ Argument $= \pi - \theta$

$= \pi - \frac{\pi}{3}$

$= \frac{3\pi-\pi}{3} = \frac{2\pi}{3}$ Ans

Q17 Find the Modulus and Argument of $-1-i$

Soln: Let $z = -1-i$

$\therefore |z| = \sqrt{(-1)^2+(-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$ Ans

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{-1}{-1}\right|$

$= \tan^{-1}|1|$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$.

$= \frac{\pi}{4}$

Here $x = -ve$ and $y = -ve$

$\therefore$ III^{rd} quadrant.

$\therefore$ Argument $= -\pi+\theta$

$= -\pi+\frac{\pi}{4}$

$= \frac{-4\pi+\pi}{4}$

$= \frac{-3\pi}{4}$ Ans.

DATE : / 20
PAGE No

Q18 Find the Modulus and Argument of $1-i$

Soln:- Let $z = 1-i$

$|z| = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$|z| = \sqrt{2}$ Ans

Arg $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{-1}{1}\right|$

$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4}$

Here $x = +ve$ and $y = -ve$

$\therefore$ IVth Quadrant.

$\therefore$ Argument $= -\theta$

$= -\frac{\pi}{4}$ Ans.

DATE: / 20
PAGE No

Q19 Find Modulus and Argument of

$a+ib$

Soln: Let $z = a+ib$

$\therefore \quad |z| = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\therefore \quad \theta = \tan^{-1}\left|\frac{b}{a}\right|$ Ans.

Q20 Find Modulus and Argument of

$\sqrt{3}+i$

Soln: Let $z = \sqrt{3}+i$

$|z| = \sqrt{(\sqrt{3})^2+(1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$\therefore \quad |z| = 2$ Ans.

$\theta = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$= \pi/6$

Here $x = \sqrt{3} = +ve$ and $y = 1 = +ve$

$\therefore$ z lies in I^{st} quadrant

DATE : / 20
PAGE No

$\therefore$ Argument $= \theta = \frac{\pi}{6}$ Ans

Q21 :- Find Modulus and Argument of $-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$

Soln :- Let $z = -\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$

$$\therefore |z| = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2} + \frac{1}{4}}$$

$$= \sqrt{\frac{6+1}{4}} = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$$

$$\therefore |z| = \frac{\sqrt{7}}{2} \text{ Ans.}$$

$$\theta = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}\right|$$

$$= \tan^{-1}\left|\frac{1}{2} \times \left(-\frac{2}{\sqrt{3}}\right)\right|$$

$$= \tan^{-1}\left|-\frac{1}{\sqrt{3}}\right|$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\tan\frac{\pi}{6}$$

$$= \frac{\pi}{6}$$

Here $x = -\frac{\sqrt{3}}{2} = -ve$, $y = \frac{1}{2} = +ve$

$\therefore$ II^{nd} quadrant.

$\therefore$ Argument $= \pi - \theta$

$= \pi - \frac{\pi}{6}$

$= \frac{6\pi - \pi}{6}$

$= \frac{5\pi}{6}$ Ans.

Q22. Convert the following complex No. in the Polar form.

$1 - i$

Soln. Let $z = 1 - i$

Note: Polar form के लिए हमें r, और θ की जरूरत होती है।

जहाँ पर $r = \sqrt{(Re(z))^2 + (Im(z))^2}$

$= \sqrt{x^2 + y^2}$

and $\theta = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{Im(z)}{Re(z)}\right|$

$r = \sqrt{(Re(z))^2 + (Im(z))^2}$

$= \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$\therefore r = \sqrt{2}$

$$\alpha = \tan^{-1}\left|\frac{Im(z)}{Re(z)}\right| = \tan^{-1}\left|\frac{y}{x}\right|$$

$$= \tan^{-1}\left|-\frac{1}{1}\right| = \tan^{-1}|-1|$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\tan\frac{\pi}{4}$$

$$\therefore \alpha = \frac{\pi}{4}$$

Here $x = 1 = +ve$, $y = -1 = -ve$

$\therefore$ IV Quadrant

$\therefore$ Argument $= -\alpha$

$$= -\frac{\pi}{4}$$

$\therefore$ Polar Form $= r[\cos\theta + i\sin\theta]$

$$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\left(\frac{\pi}{4}\right)\right]$$

Ans.

Q23 → Convert the following complex Number into Polar form

$-1-i$

DATE / / 20
PAGE No

Soln:- Let $z = -1 - i$

Here $x = -1$, and $y = -1$

$\therefore r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\therefore r = \sqrt{2}$

$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{-1}{-1}\right| = \tan^{-1}(1)$

$= \tan^{-1}\tan\frac{\pi}{4}$

$= \frac{\pi}{4}$

Here $x = -ve$ and $y = -ve$

$\underline{\text{III}^{rd}}$ Quadrant.

$\therefore$ Argument $= -\pi + \alpha$

$= -\pi + \frac{\pi}{4} = \frac{-4\pi + \pi}{4} = \frac{-3\pi}{4}$

$\therefore$ Polar Form $=$

$r(\cos\alpha + i\sin\alpha)$

$= \sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + \sin\left(-\frac{3\pi}{4}\right)\right]$

$= \sqrt{2}\left[\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right]$

Ans

Q24 :- Express the following complex Number into Polar Form.

$$2 + i2\sqrt{3}.$$

Soln:- Let $z = 2 + i2\sqrt{3}$

Here $x = 2$, $y = 2\sqrt{3}$

$\therefore$ $r = \sqrt{x^2 + y^2}$

$= \sqrt{(2)^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$

$\therefore$ $r = 4$

And $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right|$

$= \tan^{-1}(\sqrt{3}) = \cancel{\tan^{-1}}\ (\cancel{\pi}$

$\cancel{= \tan^{-1}\left(\tan\frac{\pi}{3}\right)}$

$= \frac{\pi}{4}$

Here $x = +ve$ and $y = +ve$

$\therefore$ I^{st} quadrant.

$\therefore$ Argument $= \theta = \frac{\pi}{4}$.

$\therefore$ Polar Form $= r(\cos\theta + i\sin\theta)$

$= 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ Ans

Q25 Convert $4\, cis\, 300^\circ$ to Cartesian Form.

Soln: $4\, cis\, 300^\circ = 4\cos 300^\circ + i\, 4\sin 300^\circ$

$= 4(\cos 300^\circ + i \sin 300^\circ)$

we know Polar Form of a complex No

$= r(\cos\theta + i\sin\theta)$

$\therefore r = 4, \quad \theta = 300^\circ$

$\therefore$ we know $x = r\cos\theta$

$= 4\cos 300^\circ$

$= 4\cos(360^\circ - 60^\circ)$

$= 4\cos 60^\circ$

$= 4 \times \frac{1}{2} = 2$

$= 2$

$\therefore x = 2$

And $y = r\sin\theta$

$= 4\sin 300^\circ$

$= 4\sin(360 - 60)$

$= -4\sin 60$

$= -4\frac{\sqrt{3}}{2}$

$= -2\sqrt{3}$

$\therefore y = -2\sqrt{3}$

$\therefore$ Cartesian form $z = x + iy$

$\therefore z = 2 - i2\sqrt{3}$ Ans

Note: We know complex number in cartesian form
$z = x + iy$.

And complex number in Polar Form
$z = r(\cos\theta + i\sin\theta)$

$\therefore$ $x + iy = r(\cos\theta + i\sin\theta)$

$x + iy = r\cos\theta + ir\sin\theta$

Comparing on both sides.

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

Q26 Convert the following into Rectangular or cartesian form.

$3\,\text{cis}\,0^\circ$

Soln: ~~Bet~~.

Here $3\,\text{cis}\,0^\circ = 3\cos 0^\circ + 3i\sin 0^\circ$

$= 3[\cos 0^\circ + i\sin 0^\circ]$

We know Polar Form of a complex No.

$= r[\cos\theta + i\sin\theta]$

$\therefore$ $r = 3$ and $\theta = 0^\circ$

$\therefore$ $x = r\cos\theta$
$= 3\cos 0^\circ$
$= 3 \times 1$
$= 3$

$y = r\sin\theta$
$= 3\sin 0^\circ$
$= 3 \times 0$
$y = 0$

$\therefore$ Cartesian Form

$$z = x+iy$$
$$= 3+0i$$
$$z = 3 \quad \text{Ans}$$

* Addition of two complex No.

Q27 If $z_1 = 1+3i$, $z_2 = 2+i$

Find z_1+z_2 , z_1-z_2 , $z_1 . z_2$ and $\frac{z_1}{z_2}$

Soln:-

$$z_1+z_2 = (1+3i) + (2+i)$$
$$= (1+2) + (3i+i)$$
$$= 3 + i(3+1)$$
$$= 3+4i$$

$$\therefore z_1+z_2 = 3+4i \quad \text{Ans.}$$

Subtraction of two complex No.

$$z_1 - z_2 = (1+3i) - (2+i)$$
$$= 1+3i-2-i$$
$$= (1-2) + (3i-i)$$
$$= -1 + i(3-1)$$
$$= -1+2i$$
$$\therefore z_1-z_2 = -1+2i \quad \text{Ans}$$

DATE : / 20
PAGE No

Multiplication of two complex No.

$$z_1 \cdot z_2 = (1+3i) \cdot (2+i)$$

$$= 2+i+6i+3i^2$$

$$= 2+i+6i-3$$

$$= (2-3)+i(1+6)$$

$$= -1+7i$$

$$\therefore \quad z_1 \cdot z_2 = -1+7i \quad \text{Ans.}$$

division of complex No.

$$\frac{z_1}{z_2} = \frac{1+3i}{2+i}$$

$$= \frac{1+3i}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{2-i+6i-3i^2}{(2)^2-(i)^2}$$

$$= \frac{2-i+6i-3(-1)}{4-i^2}$$

$$= \frac{2+5i+3}{4-(-1)}$$

$$= \frac{5+5i}{5}$$

$$= \frac{5}{5} + \frac{5i}{5}$$

$$= \underline{1+i} \quad \text{Ans.}$$

DATE: / 20
PAGE No

Q28 If $z_1 = 1+3i$, $z_2 = 4+7i$

Find $z_1 - z_2 = (1+3i) - (4+7i)$

$= 1+3i-4-7i$

$= (1-4) + (3i-7i)$

$= -3 + i(3-7)$

$= -3 + i(-4)$

$= -3-4i$ Ans.

Q29 If $z_1 = 3-6i$, $z_2 = 2-3i$ Find z_1+z_2.

Soln:- $z_1+z_2 = (3-6i) + (2-3i)$

$= 3-6i+2-3i$

$= 5-9i$ Ans.

Q30 If $z_1 = 1+3i$, $z_2 = 2+i$ Find z_1+z_2

Soln:- $z_1+z_2 = 1+3i+2+i$

$= 3+4i$ Ans.

Q31:- If $z_1 = 1+3i$, $z_2 = 2+i$ Find $z_1 . z_2$

Soln:- $z_1 . z_2 = (1+3i) . (2+i)$

$= 2+i+6i+3i^2$

$= 2+7i-3$

$= -1+7i$ Ans.

DATE: / 20
PAGE No

Q32 Simplify $(5+3i)(4-i)$

Soln:- $(5+3i)(4-i) = 20-5i+12i-3i^2$

$= 20+7i-3(-1)$

$= 20+7i+3$

$= 23+7i$ Ans.