* Matrices and Determinants

*** Define Matrix :-** The arrangement of numbers (object) in rows and column is called a Matrix.

e.g $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

*** Elements of a Matrix :-** The Numbers which are used in Matrix are called elements of a Matrix.

e.g $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Here 1, 2, 3 and 4 are elements of a Matrix.

*** Rows of a Matrix :-** Horizontal arrangement of the elements are called rows of the Matrix.

e.g $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Here $[1 \ 2]$ and $[3 \ 4]$ are Rows of Matrix.

*** Column of a Matrix :-** Vertical arrangement of the elements are called columns of Matrix.

e.g $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Here $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ are columns of a Matrix.

* **Define order of a Matrix :-**

The combination of number of rows and columns in a matrix are called order of Matrix.
We Represent order of Matrix by $m \times n$.
where
m = number of Rows in a Matrix
n = number of Columns in a Matrix

e.g $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$

it is a 2×2 order Matrix.

* **Types of Matrics :-**

1. **Row Matrix :-**

A Matrix which has only single Row is called Row Matrix.

eg $A = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1 \times 2}$

2. **Column Matrix:**

A Matrix which has only one column is called column Matrix.

Ex. $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$

3. **Rectangular Matrix:**

The Matrix in which number of Rows not equal to number of columns is called Rectangular Matrix.

e.g $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

4. **Square Matrix:**

The Matrix in which Number of Row equal to Number of columns is called Square Matrix.

e.g $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

5. **Diagonal Matrix:**

A Square Matrix in which ~~diagonal elements are~~ all entries are zero except diagonals Elements is called diagonal Matrix.

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e.g. $A = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}_{2\times2}$

6. **Scaler Matrix:-**

A diagonal Matrix in which all diagonal elements are same is called scaler Matrix.

e.g. $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times2}$

7. **Identity Matrix/ UNIT Matrix:-**

A diagonal Matrix in which all diagonal elements are equal to '1' is called Identity Matrix.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

8. **Zero Matrix:-**

A matrix in which all elements are zero is called zero Matrix.

e.g. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}$

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9:- Upper Triangular Matrix → A Square Matrix in which all elements below principal diagonal elements are zero is called upper Triangular Matrix.

eg $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}_{3\times3}$ Principal diagonal

10. Lower Triangular Matrix → A Square Matrix in which all elements above principal diagonal elements are zero is called Lower Triangular Matrix.

e.g $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}_{3\times3}$

Principal diagonal.

11 Equal Matrices :- → Two Matrices are equal

(1) if they are of same order.

(2) If their corresponding elements are equal.

e.g $A = \begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix}_{2\times2}$, $B = \begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix}_{2\times2}$

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Q1:- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{2\times 2}$

Find A+B , A−B

Soln:- $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} 1+3 & 2+4 \\ 3+5 & 4+6 \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}_{2\times 2}$ Ans.

$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} - \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} 1-3 & 2-4 \\ 3-5 & 4-6 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$ Ans.

Q2 If $A = \begin{bmatrix} 5 & 4 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$

Find 3A+4B

Soln:- $3A + 4B = 3\begin{bmatrix} 5 & 4 \\ 2 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$

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$$= \begin{bmatrix} 3\times5 & 3\times4 \\ 3\times2 & 3\times(-3) \end{bmatrix} + \begin{bmatrix} 4\times1 & 4\times0 \\ 4\times(-2) & 4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 12 \\ 6 & -9 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -8 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 15+4 & 12+0 \\ 6-8 & -9+12 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 12 \\ -2 & 3 \end{bmatrix}$$ Ans

Q3 If $A = \begin{bmatrix} 2 & 3 \\ -5 & 7 \end{bmatrix}$, and $B = \begin{bmatrix} -2 & 4 \\ 5 & 8 \end{bmatrix}$

Then Find $A-B$.

Soln: $A - B = \begin{bmatrix} 2 & 3 \\ -5 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 5 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 2-(-2) & 3-4 \\ -5-5 & 7-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -1 \\ -10 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 \\ -10 & -1 \end{bmatrix}$$ Ans

Q4: If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

Find $2A-3B$

Soln:

$$2A-3B = 2\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 5 & 2\times 3 \\ 2\times(-1) & 2\times 1 \end{bmatrix} - \begin{bmatrix} 3\times 2 & 3\times(-1) \\ 3\times 3 & 3\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ 9 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10-6 & 6-(-3) \\ -2-9 & 2-6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ -11 & -4 \end{bmatrix} \text{ Ans.}$$

Q5 If $A = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$

Find $A+2B$.

Soln:

$$A+2B = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix} + 2\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 2\times(-3) & 2\times 2 \\ 2\times(-2) & 2\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-6 & 1+4 \\ 5-4 & 1+2 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 1 & 3 \end{bmatrix}$$

Ans

Q6:- If $X = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$

Find $3X + Y$.

Soln:- $3X + Y = 3\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3\times1 & 3\times2 \\ 3\times(-3) & 3\times4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3+4 & 6+5 \\ -9+1 & 12-3 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$ Ans

Q7:- If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

(i) Find $2A + 3B - 4I$ where I is unit Matrix

(ii) Find $3A - 2B$

Soln:- $2A + 3B - 4I = 2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\times2 & 2\times3 \\ 2\times4 & 2\times7 \end{bmatrix} + \begin{bmatrix} 3\times1 & 3\times3 \\ 3\times4 & 3\times6 \end{bmatrix} - \begin{bmatrix} 4\times1 & 4\times0 \\ 4\times0 & 4\times1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

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$$= \begin{bmatrix} 4+3-4 & 6+9-0 \\ 8+12-0 & 14+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 15 \\ 20 & 28 \end{bmatrix}$$ Ans

(ii) $3A - 2B = 3\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 3\times 2 & 3\times 3 \\ 3\times 4 & 3\times 7 \end{bmatrix} - \begin{bmatrix} 2\times 1 & 2\times 3 \\ 2\times 4 & 2\times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-2 & 9-6 \\ 12-8 & 21-12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ 4 & 9 \end{bmatrix}$$ Ans

Q8:- If $A = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$

Find AB and show AB is a Null Matrix

Soln:- $AB = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$

$$= \begin{bmatrix} 4\times 2 + 2\times(-4) & 4\times 6 + 2\times(-12) \\ 8\times 2 + 4\times(-4) & 8\times 6 + 8\times(-12) \end{bmatrix}$$

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$$AB = \begin{bmatrix} 8-8 & 24-24 \\ 16-16 & 48-48 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{Null Matrix}$$

Q9:- Compute $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$= \begin{bmatrix} a\times a + b\times b & a\times(-b) + b\times a \\ -b\times a + a\times b & -b\times(-b) + a\times a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix} \text{ Ans.}$$

Q10:- If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Show that $AB = BA = I$

Soln:- $AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2\times3 + 5\times(-1) & 2\times(-5)+5\times2 \\ 1\times3+3\times(-1) & 1\times(-5)+3\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad —①$$

$$BA = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times2+(-5)\times1 & 3\times5+(-5)\times3 \\ -1\times2+2\times1 & -1\times5+2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & 15-15 \\ -3+3 & -5+6 \end{bmatrix}$$

$$\therefore \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad —②$$

From ① and ② we have

$AB = BA = I$

Hence Proof

Q 11:- If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$

Then Evaluate $AB + 2I$

Soln:- $AB + 2I = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2\times(-1)+1\times4 & 2\times(-2)\times1\times5 \\ 3\times(-1)+4\times4 & 3\times(-2)+4\times5 \end{bmatrix} + \begin{bmatrix} 2\times1 & 2\times0 \\ 2\times0 & 2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4 & -4+5 \\ -3+16 & -6+20 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$\therefore AB + 2I = \begin{bmatrix} 2+2 & 1+0 \\ 13+0 & 14+2 \end{bmatrix}$

$= \begin{bmatrix} 4 & 1 \\ 13 & 16 \end{bmatrix}$ Ans.

Q12. If $A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$

Show that $AB = BA$

$AB = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2\times4 + 3\times1 & 2\times(-1) + 3\times4 \\ -3\times4 + 2\times1 & -3\times(-1) + 2\times4 \end{bmatrix}$

$= \begin{bmatrix} 8+3 & -2+12 \\ -12+2 & 3+8 \end{bmatrix}$

$AB = \begin{bmatrix} 11 & 10 \\ -10 & 11 \end{bmatrix}$ — (1)

$BA = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4\times2 + (-1)\times(-3) & 4\times3 + (-1)\times2 \\ 1\times2 + 4\times(-3) & 1\times3 + 4\times(2) \end{bmatrix}$

$= \begin{bmatrix} 8+3 & 12-2 \\ 2-12 & 3+8 \end{bmatrix}$

$= \begin{bmatrix} 11 & 10 \\ -10 & 11 \end{bmatrix}$ — (2)

From ① and ② we have

$$AB = BA.$$

* Transpose of a Matrix :-

If rows and columns of matrix A are interchanged, then the new Matrix thus obtained is called the Transpose of A. and it is denoted by $A^t = A'$.

e.g $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

$\therefore A^t = A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}'_{2\times 2}$

$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times 2}$ Ans.

* Symmetric Matrix:-

A Square Matrix A is said to be symmetric if it is equal to its transpose ie, $A = A^t$

e.g $A = \begin{bmatrix} 1 & 4 \\ 4 & 3 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 4 \\ 4 & 3 \end{bmatrix}' = \begin{bmatrix} 1 & 4 \\ 4 & 3 \end{bmatrix}.$

* Skew Symmetric Matrix:- A square Matrix A is said to be skew symmetric if $A^t = -A$.

e.g $A = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$

$\therefore A' = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}'$

$= \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} = -A$

* Determinant:- To each square Matrix we associate a number and that number is called determinant of given Matrix.

e.g $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$= (1) \times (4) - (2) \times (3)$

$= 4 - 6$

$= -2$

Q13) Evaluate $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{vmatrix}$

Soln) $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{vmatrix} = (\sin\alpha)\times(\sin\alpha) - (\cos\alpha)\times(-\cos\alpha)$

$= \sin^2\alpha + \cos^2\alpha$

$= 1$ Ans.

Q14) Evaluate $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

Soln) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = (1)\times(0) - (1)\times(1)$

$= 0 - 1$

$= -1$ Ans

Q15 Evaluate $\begin{vmatrix} 3 & 9 \\ 1 & 6 \end{vmatrix}$

Soln) $\begin{vmatrix} 3 & 9 \\ 1 & 6 \end{vmatrix} = (3)\times(6) - (1)\times(9)$

$= 18 - 9$

$= 9$ Ans.

Q16) Find x if $\begin{vmatrix} 4 & x \\ x & 4 \end{vmatrix} = 0$

Soln) $\begin{vmatrix} 4 & x \\ x & 4 \end{vmatrix} = 0$

$\Rightarrow 4\times 4 - x.x = 0$

$$16 - x^2 = 0$$
$$\Rightarrow +x^2 = +16$$
$$\Rightarrow x^2 = 16$$
$$\Rightarrow x = \pm 4 \quad \text{Ans}$$

Q17 If $\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$

then find the value of x.

Soln-
$$\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$$
$$\Rightarrow (3) \times (4) - (x) \times (2) = 0$$
$$\Rightarrow 12 - 2x = 0$$
$$\Rightarrow +2x = +12$$
$$\Rightarrow x = \frac{12}{2}$$
$$\Rightarrow x = 6 \quad \text{Ans.}$$

Q18 Evaluate $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Soln-
$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = (2) \times (-1) - (4) \times (-5)$$
$$= -2 + 20$$
$$= 18 \quad \text{Ans}$$

Q19 Evaluate $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Soln-
$$\begin{vmatrix} a & c \\ b & d \end{vmatrix} = (a) \times (d) - (b) \times (c)$$
$$= ad - bc$$
Ans

Q20 Evaluate $\begin{vmatrix} \sec\alpha & \tan\alpha \\ \tan\alpha & \sec\alpha \end{vmatrix}$

Soln:- $\Rightarrow \begin{vmatrix} \sec\alpha & \tan\alpha \\ \tan\alpha & \sec\alpha \end{vmatrix}$

$= (\sec\alpha) \times (\sec\alpha) - (\tan\alpha)(\tan\alpha)$

$= \sec^2\alpha - \tan^2\alpha$

$= 1$ Ans.

Q21 Evaluate $\begin{vmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{vmatrix}$

Soln:- $\begin{vmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{vmatrix} = (1)\times(1) - (\sin\alpha) \times (\sin\alpha)$

$= 1 - \sin^2\alpha$

$= \cos^2\alpha.$ Ans

Q22 Find value of x, if

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Soln:- $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ Given

$\therefore (2) \times (5) - (4) \times (3) = (x) \times (5) - (2x) \times (3)$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow \cancel{-}2 = \cancel{-}x$

$\Rightarrow x = 2$ Ans

Q23 Find value of x if

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

Soln:- $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ Given

$\therefore \quad (x) \times (x) - (18) \times (2) = (6) \times (6) - (18) \times (2)$

$\Rightarrow \quad x^2 - 36 = 36 - 36$

$\Rightarrow \quad x^2 - 36 = 0$

$\Rightarrow \quad x^2 = 36$

$\Rightarrow \quad x = \pm\sqrt{36}$

$\Rightarrow \quad x = \pm 6$ Ans

Q24:- Evaluate $\begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 6 \end{vmatrix}$

Soln:- $\begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 6 \end{vmatrix} =$

$3 \begin{vmatrix} 1 & 0 \\ 0 & 6 \end{vmatrix} - 0 \begin{vmatrix} 2 & 0 \\ 3 & 6 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix}$

$= 3(6 - 0) - 0(12 - 0) + 1(0 - 3)$

$= 18 - 0 - 3$

$= 15$ Ans

Q25 Evaluate $\begin{vmatrix} 2 & 3 & 6 \\ 0 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix}$

Soln:- $\begin{vmatrix} 2 & 3 & 6 \\ 0 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} - 3\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix} + 6\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}$

$= 2(3-0) - 3(0-4) + 6(0-2)$

$= 6 + 12 - 12$

$= 6$ Ans.

Q26 Evaluate $\begin{vmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ -1 & 2 & 4 \end{vmatrix}$

Soln:- $\begin{vmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ -1 & 2 & 4 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$

$= 3(4-6) - 4(8-(-3)) + 2(4-(-1))$

$= 3(-2) - 4(8+3) + 2(4+1)$

$= -6 - 44 + 10$

$= -40$ Ans.

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Q27. Evaluate $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Soln:- $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

$= 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$= 3(1-(-6)) + 4(1-(-4)) + 5.(3-2)$

$= 3(1+6) + 4(1+4) + 5(1)$

$= 21 + 20 + 5$

$= 46$ Ans.

Q28 Evaluate $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Soln:- $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 2 \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$

$= 2(0-(-1)(-5)) + 1(0-(-3)) - 2(0-6)$

$= 2(0-5). + 1(0+3) - 2(0-6)$

$= -10 + 3 + 12$

$= 5$ Ans.

Q29 Solve the following equations by Cramer's Rule

$x + 3y = 4$, $4x - y = 3$

Soln: Given Equations are

$x + 3y = 4$ —①
$4x - y = 3$ —②

$\therefore \quad \Delta = D = \begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix}$

$= (1) \times (-1) - (3) \times (4)$
$= -1 - 12 = -13$

$\Delta_1 = D_1 = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix}$

$= (4) \times (-1) - (3) \times (3)$
$= -4 - 9 = -13$

$\Delta_2 = D_2 = \begin{vmatrix} 1 & 4 \\ 4 & 3 \end{vmatrix}$

$= (1) \times (3) - (4) \times (4)$
$= 3 - 16 = -13$

$\therefore$ By Cramer's Rule

$x = \frac{\Delta_1}{\Delta} = \frac{D_1}{D} = \frac{-13}{-13} = 1$ Ans

$y = \frac{\Delta_2}{\Delta} = \frac{D_2}{D} = \frac{-13}{-13} = 1$ Ans

Q3. Solve the following equations by cramer's Rule
$3x+4y=-14$, $-2x-3y=11$

Soln: Given Equations are

$$3x+4y=-14 \quad —(1)$$
$$-2x-3y=11 \quad —(2)$$

$$\therefore \quad \Delta = D = \begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}$$

$$= (3)\times(-3) - (-2)\times(4)$$
$$= -9+8 = -1$$

$$\Delta_1 = D_1 = \begin{vmatrix} -14 & 4 \\ 11 & -3 \end{vmatrix}$$

$$= (-14)\times(-3) - (4)\times(11)$$
$$= 42-44 = -2$$

$$\Delta_2 = D_2 = \begin{vmatrix} 3 & -14 \\ -2 & 11 \end{vmatrix}$$

$$= (3)\times(11) - (-2)\times(-14)$$
$$= 33-28$$
$$= 5$$

$\therefore$ By Cramer's Rule

$$x = \frac{\Delta_1}{\Delta} = \frac{D_1}{D} = \frac{-2}{-1} = 2 \text{ Ans}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{D_2}{D} = \frac{5}{-1} = -5 \text{ Ans}$$

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Q.31 Solve the following Equations by Cramer's Rule

$-x+5y=2$, $x-2y=-2$

Soln:- Given Equations are

$-x+5y=2$ — (1)

$x-2y=-2$ — (2)

$\therefore$ $D = \begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix}$

$= (-1)\times(-2) - (1)\times(5)$

$= 2-5 = -3$

$D_1 = \begin{vmatrix} 2 & 5 \\ -2 & -2 \end{vmatrix}$

$= (2)\times(-2) - (-2)\times(5)$

$= -4+10 = 6$

$D_2 = \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix}$

$= (-1)\times(-2) - (1)\times(2)$

$= 2-2 = 0$

$\therefore$ By Cramer's Rule

$x = \frac{D_1}{D} = \frac{6}{-3} = -2$ Ans

$y = \frac{D_2}{D} = \frac{0}{-3} = 0$ Ans

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Q8) Solve the following Equation by cramer's Rule

$2x-y=-2$ and $3x+4y=3$

Soln:- Given Equations are

$2x-y=-2$ —— (1)

$3x+4y=3$ —— (2)

$$\therefore \quad D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= (2)\times(4)-(3)\times(-1)$$

$$= 8+3 = 11$$

$$D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= (-2)\times(4)-(-1)\times(3)$$

$$= -8+3$$

$$= -5$$

$$D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix}$$

$$= (2)\times(3)-(-2)\times(3)$$

$$= 6+6$$

$$= 12$$

$\therefore$ By Cramer's Rule

$$x = \frac{D_1}{D} = \frac{-5}{11}$$

$$y = \frac{D_2}{D} = \frac{12}{11}$$

Ans

Q33:- Solve the following Equation by cramer's Rule
$2x-3y+2z=5$, $4x+2y-3z=2$
and $x-2y+5z=3$.

Soln:- Given Equations are

$2x-3y+2z=5$ —(1)

$4x+2y-3z=2$ —(2)

$x-2y+5z=3$ —(3)

$$\therefore \Delta = D = \begin{vmatrix} 2 & -3 & 2 \\ 4 & 2 & -3 \\ 1 & -2 & 5 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & -3 \\ -2 & 5 \end{vmatrix} + 3\begin{vmatrix} 4 & -3 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix}$$

$= 2(10-6)+3(20+3)+2(-8-2)$

$= 2\times4+3\times23+2\times(-10)$

$= 8+69-20$

$= +57$

$$\Delta_1 = D_1 = \begin{vmatrix} 5 & -3 & 2 \\ 2 & 2 & -3 \\ 3 & -2 & 5 \end{vmatrix}$$

$$\Rightarrow 5\begin{vmatrix} 2 & -3 \\ -2 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix}$$

$= 5(10-6)+3(10+9)+2(-4-6)$

$= 5\times4+3\times19+2\times(-10)$

$= 20+57-20$

$= 57$

$$\Delta_2 = D_2 = \begin{vmatrix} 2 & 5 & 2 \\ 4 & 2 & -3 \\ 1 & 3 & 5 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix} - 5\begin{vmatrix} 4 & -3 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

$= 2(10+9) - 5(20+3) + 2(12-2)$

$= 2 \times 19 - 5 \times 23 + 2 \times 10$

$= 38 - 115 + 20$

$= -57$

$$\therefore \Delta_3 = D_3 = \begin{vmatrix} 2 & -3 & 5 \\ 4 & 2 & 2 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 2 \\ -2 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} + 5\begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix}$$

$= 2(6+4) + 3(12-2) + 5(-8-2)$

$= 2 \times 10 + 3 \times 10 + 5 \times (-10)$

$= 20 + 30 - 50$

$= 0$

$\therefore$ By Cramer's Rule

$x = \dfrac{\Delta_1}{\Delta} = \dfrac{D_1}{D} = \dfrac{57}{57} = 1$

$y = \dfrac{\Delta_2}{\Delta} = \dfrac{D_2}{D} = \dfrac{-57}{57} = -1$

$z = \dfrac{\Delta_3}{\Delta} = \dfrac{D_3}{D} = \dfrac{0}{57} = 0$

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Q34) Solve the following system of equation by cramer's / determinant method.

$3x - 4y - z = 2$, $x + 6y + 3z = 7$

and $9x - 8y - 5z = 0$

Soln Given Equations are

$3x - 4y - z = 2$ — (1)

$x + 6y + 3z = 7$ — (2)

$9x - 8y - 5z = 0$ — (3)

$$\therefore \quad D = \begin{vmatrix} 3 & -4 & -1 \\ 1 & 6 & 3 \\ 9 & -8 & -5 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 3 \\ -8 & -5 \end{vmatrix} + 4\begin{vmatrix} 1 & 3 \\ 9 & -5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 6 \\ 9 & -8 \end{vmatrix}$$

$= 3(-30+24) + 4(-5-27) - (-8-54)$

$= 3(-6) + 4(-32) - (-62)$

$= -18 - 128 + 62$

$= -84$

$$D_1 = \begin{vmatrix} 2 & -4 & -1 \\ 7 & 6 & 3 \\ 0 & -8 & -5 \end{vmatrix}$$

$$= 2\begin{vmatrix} 6 & 3 \\ -8 & -5 \end{vmatrix} + 4\begin{vmatrix} 7 & 3 \\ 0 & -5 \end{vmatrix} + (-1)\begin{vmatrix} 7 & 6 \\ 0 & -8 \end{vmatrix}$$

$= 2(-30+24) + 4(-35-0) - 1(-56-0)$

$= 2(-6) + 4(-35) - 1(-56)$

$= -12 - 140 + 56$

$= -96$

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$$D_2 = \begin{vmatrix} 3 & 2 & -1 \\ 1 & 7 & 3 \\ 9 & 0 & -5 \end{vmatrix}$$

$$= 3\begin{vmatrix} 7 & 3 \\ 0 & -5 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 9 & -5 \end{vmatrix} - 1\begin{vmatrix} 1 & 7 \\ 9 & 0 \end{vmatrix}$$

$$= 3(-35-0) - 2(-5-27) - 1(0-63)$$
$$= 3(-35) - 2(-32) - 1(-63)$$
$$= -105 + 64 + 63$$
$$= -105 + 127$$
$$= 22$$

$$D_3 = \begin{vmatrix} 3 & -4 & 2 \\ 1 & 6 & 7 \\ 9 & -8 & 0 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 7 \\ -8 & 0 \end{vmatrix} + 4\begin{vmatrix} 1 & 7 \\ 9 & 0 \end{vmatrix} + 2\begin{vmatrix} 1 & 6 \\ 9 & -8 \end{vmatrix}$$

$$= 3(0+56) + 4(0-63) + 2(-8-54)$$
$$= 3(56) + 4(-63) + 2(-62)$$
$$= 168 - 252 - 124$$
$$= -208$$

$\therefore$ By cramer's Rule

$$x = \frac{D_1}{D} = \frac{+96}{+84} = \frac{96}{84} = \frac{48}{42} = \frac{24}{21} = \frac{8}{7}$$

$$y = \frac{D_2}{D} = \frac{22}{84} = \frac{11}{42}$$

$$z = \frac{D_3}{D} = \frac{-208}{+84} = \frac{-52}{21}$$

Ans

Q35 Apply Cramer's Rule or Determinant Rule to solve the following system of linear equations

$$x+y+z=-1$$

$x+2y+3z=-4$ and $x+3y+4z=-6$

Soln: Given System of equations are

$$x+y+z=-1 \quad —①$$
$$x+2y+3z=-4 \quad —②$$
$$x+3y+4z=-6 \quad —③$$

Soln:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= 1(8-9) - 1(4-3) + 1(3-2)$$
$$= -1 - 1 + 1$$
$$= -1$$

$$D_1 = \begin{vmatrix} -1 & 1 & 1 \\ -4 & 2 & 3 \\ -6 & 3 & 4 \end{vmatrix}$$

$$= -1\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ -6 & 3 \end{vmatrix}$$

$$= -1(8-9) - 1(-16+18) + 1(-12+12)$$
$$= -1(-1) - 1(2) + 0$$
$$= 1-2$$
$$= -1$$

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$$D_2 = \begin{vmatrix} 1 & -1 & 1 \\ 1 & -4 & 3 \\ 1 & -6 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & -4 \\ 1 & -6 \end{vmatrix}$$

$= 1(-16+18) + 1(4-3) + 1(-6+4)$

$= 2 + 1 - 2$

$= 1$

$$D_3 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 1 & 3 & -6 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} - 1\begin{vmatrix} 1 & -4 \\ 1 & -6 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$= 1(-12+12) - 1(-6+4) - 1(3-2)$

$= 0 + 2 - 1$

$= 1$

$\therefore$ By Cramer's Rule

$x = \frac{D_1}{D} = \frac{-1}{-1} = 1$

$y = \frac{D_2}{D} = \frac{1}{-1} = -1$

$z = \frac{D_3}{D} = \frac{1}{-1} = -1$

Q 36 If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Prove that $A^2 - 7A - 2I = 0$

Soln :- Let :- $A^2 - 7A - 2I$

$= A \cdot A - 7 \cdot A - 2I$

$= \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - 7 \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\times2 + 3\times4 & 2\times3 + 3\times5 \\ 4\times2 + 5\times4 & 4\times3 + 5\times5 \end{bmatrix} - \begin{bmatrix} 14 & 21 \\ 28 & 35 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+12 & 6+15 \\ 8+20 & 12+25 \end{bmatrix} - \begin{bmatrix} 14 & 21 \\ 28 & 35 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix} - \begin{bmatrix} 14 & 21 \\ 28 & 35 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 11 - 14 - 2 & 21 - 21 - 0 \\ 28 - 28 - 0 & 37 - 35 - 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$=$ R.H.S.

Q 37 If $f(x) = x^2 - 4x + 1$ Find $f(A)$

where $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

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Soln:- If $f(x) = x^2 - 4x + 1$

$\therefore f(A) = A^2 - 4A + I$

$= A \cdot A - 4A + I$

$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\times2+3\times1 & 2\times3+3\times2 \\ 1\times2+2\times1 & 1\times3+2\times2 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Ans