G.P. Dhangar

# Fluid Mechanics

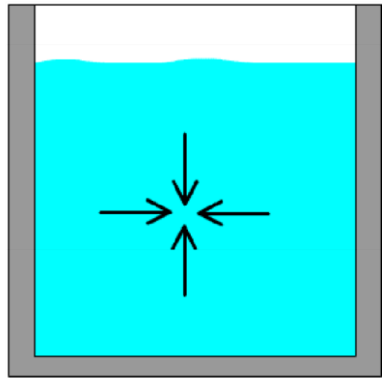
By
Er. Vijaypal
Deptt. Of Civil Engg.

# Pressure

- Pressure is the (compression) force exerted by a fluid per unit area.

$$Pressure = \frac{Force}{Area} \quad \left(\frac{N}{m^2}\right) \equiv Pa$$

- Stress vs. pressure?
- In fluids, gases and liquids, we speak of pressure; in solids this is normal stress. For a fluid at rest, the pressure at a given point is the same in all directions.



- Differences or gradients in pressure drive a fluid flow, especially in ducts and pipes.

# Density

The density of a fluid is its mass per unit volume:

$$\rho = \frac{m}{V} \quad \left(\frac{kg}{m^3}\right)$$

- Liquids are essentially incompressible
- Density is highly variable in gases nearly proportional to the pressure.

| @20˚C, 1 atm | Air | Water | Hydrogen | Mercury |
|---|---|---|---|---|
| Density (kg/m$^3$) | 1.20 | 998 | 0.0838 | 13,580 |

- **Note**: *specific volume* is defined as:

$$v = \frac{V(m^3)}{m(kg)} = \frac{1}{\rho}$$

# Specific weight

- The specific weight of a fluid is its weight, , per unit volume. Density and specific weight are related by gravity:

$$\gamma = \rho g \quad \left(\frac{N}{m^3}\right)$$

# Specific gravity

Specific gravity is the ratio of a fluid density to a standard reference fluid, typically water at 4˚C (for liquids) and air (for gases):

$$SG_{gas} = \frac{\rho_{gas}}{\rho_{air}} = \frac{\rho_{gas}}{1.205\ (kg/m^3)}$$

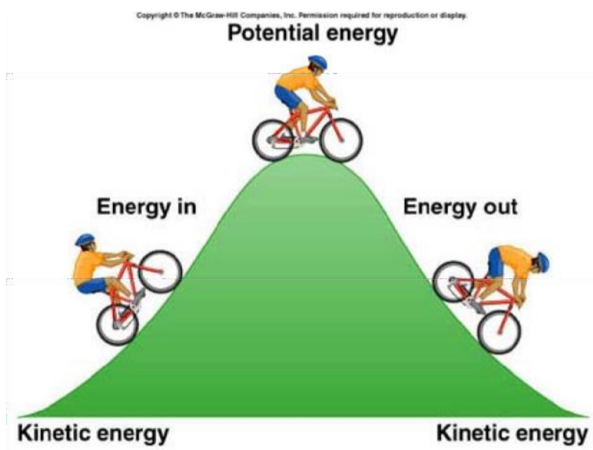
$$SG_{liquid} = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\rho_{liquid}}{1000\ (kg/m^3)}$$

- For example, the specific gravity of mercury is $SG_{Hg}$= 13,580/1000 ٧٣13.6.

# Kinetic and potential energy

- <u>Potential energy</u> is the work required to move the system of mass *m* from the origin to a position against a gravity field *g*:

$$PE = mgz$$



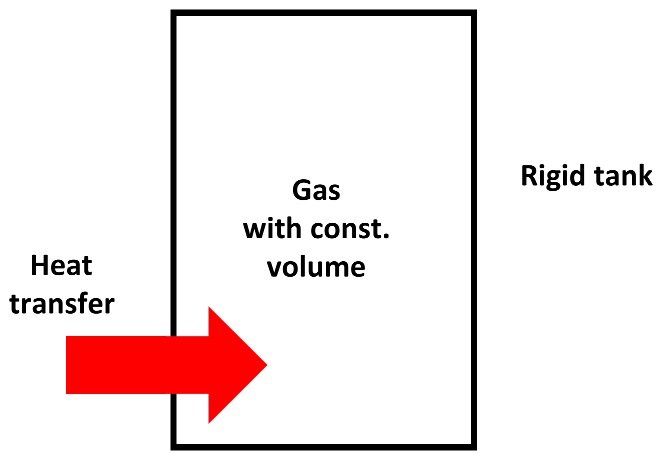
- Kinetic energy is the work required to change the speed of the mass from zero to velocity *V*.

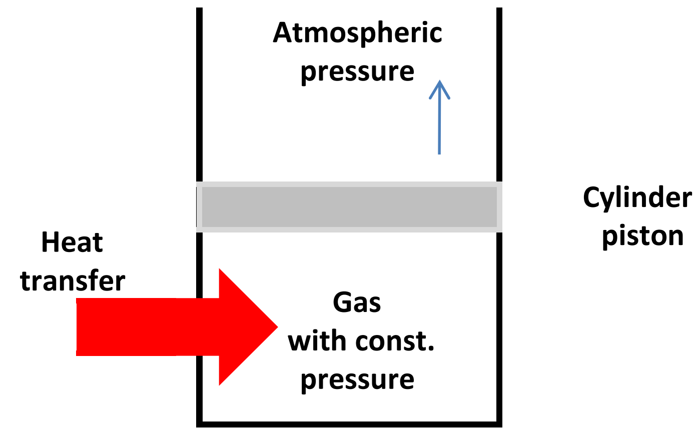
$$KE = \frac{1}{2}mV^2$$

Note: internal energy, *u*, is a function of temperature and pressure for the single-phase substance, whereas KE and PE are kinematic quantitie

# Specific heat

- Specific heat capacity: is the measure of the heat energy required to increase the temperature of a unit mass of a substance by one degree temperature.
- $c_{aluminum}$=0.9 (kJ/kg.K) and $c_{water}$= 4.186 ( kJ/kg.K )
- There are two types of specific heats, constant volume $c_v$and constant pressure $c_p$.

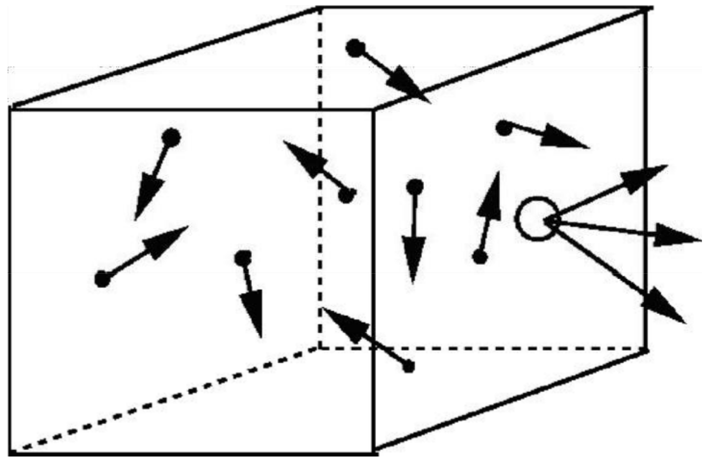



# $c_v$ $c_p$ Ideal gas equation of state

- Any equation that relates the pressure, temperature, and specific volume of a substance is called an equation of state.
- It is experimentally observed that at a low pressure the volume of a gas is

proportional to its temperature:



$$p = \rho R T \quad p = R_u \rho T$$

$R_u$ is the gas universal constant, $R_u$= 8.314 (*kJ/kmol.K*)

The constant *R* is different for each gas; for air, $R_{air}$= 0.287 (*kJ/kg.K)*. The molecular weight of air *M*=28.97 *kg/kmol*.

# Properties of ideal gas

- For an ideal gas, *internal energy* is only a function of temperature; thus constant volume specific heat is only a function of temperature:

$$c_v = \left(\frac{\partial u}{\partial T}\right)\Big|_v = \frac{du}{dT} = c_v(T)$$

$$du = c_v(T)dT$$

- For an ideal gas, *enthalpy* is only a function of temperature; *h= u + pv*• The constant pressure specific heat can be defined as:

$$c_p = \left(\frac{\partial h}{\partial T}\right)\Big|_p = \frac{dh}{dT} = c_p(T)$$

$$dh = c_p(T)dT$$

$$R = c_p - c_v$$

- The specific heat ratio is an important dimensionless parameter:

$$k = \frac{c_p}{c_v} = k(T) \geq 1$$

# Incompressible fluids

- Liquids are (almost) incompressible and thus have a single constant specific heat:

$$c_p = c_v = c \qquad dh = c\,dT$$

# Viscosity

- Viscosity is a measure of a fluid's resistance to flow. It determines the fluid strain rate that is generated by a given applied shear stress.

- Temperature has a strong and pressure has a moderate effect on viscosity. The viscosity of gases and most liquids increases slowly with pressure.

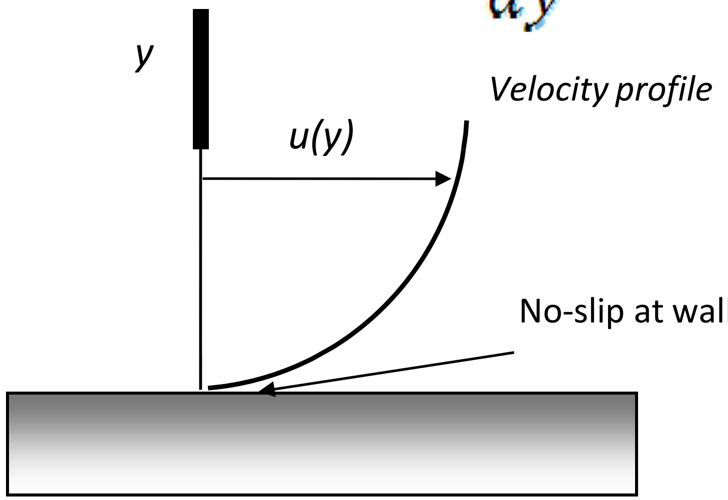
$$\mu_{hydrogen} = 9.0E-6\ \left(\frac{kg}{m.s}\right) \qquad \mu_{air} = 1.8E-5\ \left(\frac{kg}{m.s}\right)$$

$$\mu_{water} = 1.0E-3\ \left(\frac{kg}{m.s}\right) \qquad \mu_{engine\ oil, SAE30} = 0.20\ \left(\frac{kg}{m.s}\right)$$

# Viscosity

- A <u>Newtonian fluid</u> has a linear relationship between shear stress and

$$\tau = \mu \frac{du}{dy}$$




velocity gradient:

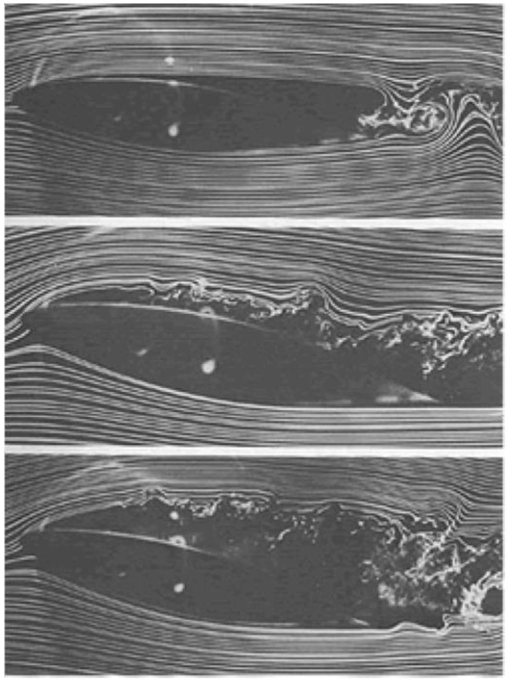
- The *no-slip condition*: at the wall velocity is zero relative to the wall. This is a characteristic of all viscous fluid.

- The shear stress is proportional to the slope of the velocity profile and is greatest at the wall.

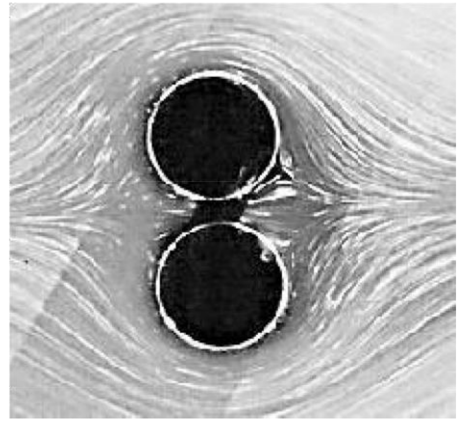
# The Reynolds number

- The Reynolds number, *Re*, is a dimensionless number that gives a measure of

$$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$



the ratio of inertial forces to viscous forces



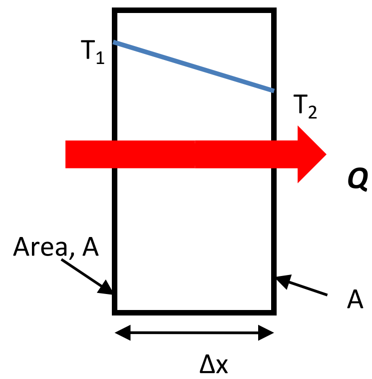
Creeping flow, Re is very low

Laminar flow, Re moderate
Turbulent flow, Re high

# Thermal conductivity

- Rate of heat conduction is proportional to the temperature difference, but it is inversely proportional to the thickness of the layer

$$Rate\ of\ heat\ transfer \propto \frac{(surface\ area)(temperature\ difference)}{wall\ thickness}$$



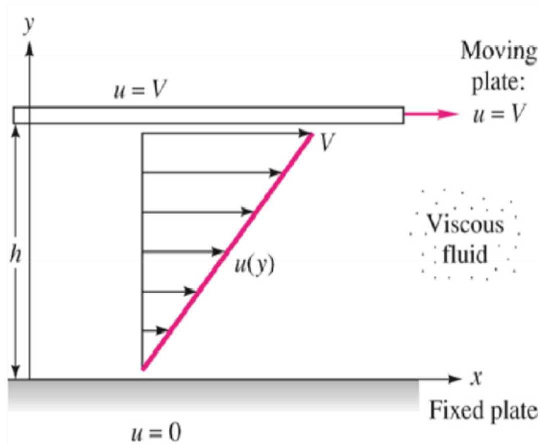

- To make this equality, *k* (*W/m.K*)the thermal conductivity of the material, is introduced.
- This is called the Fourier's law of heat conduction:

$$q = \frac{Q}{A} = -k\nabla T$$

$$q_x = -k\frac{\partial T}{\partial x}, \quad q_y = -k\frac{\partial T}{\partial y}, \quad q_z = -k\frac{\partial T}{\partial z}$$

# Flow between parallel plates

- It is the flow induced between a fixed lower plate and upper plate moving steadily at velocity *V*




- Shear stress is constant throughout the fluid:

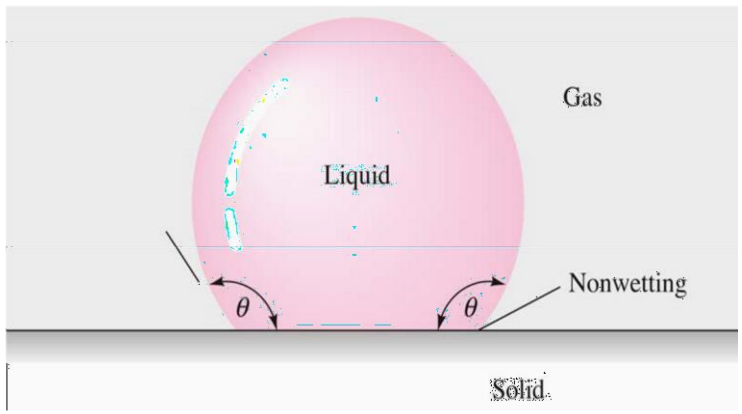
$$\frac{du}{dy} = \frac{\tau}{\mu} = const.$$

- After integration and applying boundary conditions:

$u = V\frac{y}{h}$

# Surface tension

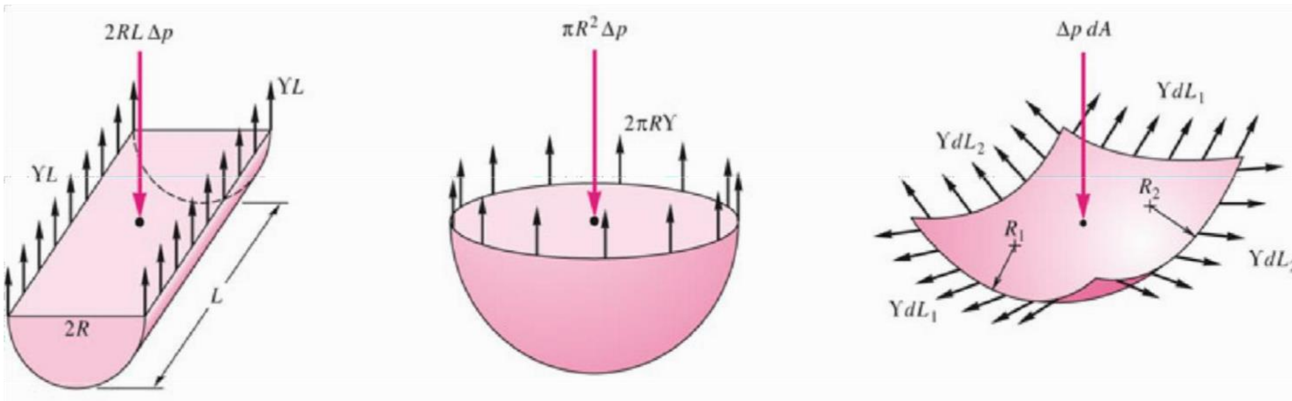
- A liquid, being unable to expand freely, will form an *interface* with a second liquid or gas.




- The cohesive forces between liquid molecules are responsible for the phenomenon known as surface tension.

- Surface tension *Υ* (pronounced upsilon) has the dimension of force per unit length (*N/m*) or of energy per unit area (*J/m²*).
- $\Upsilon_{air\text{-}water} = 0.073$ N/m; $\Upsilon_{air\text{-}mercury} = 0.48$ N/m

# Surface tension



(a) (b) (c)

- Using a force balance, pressure increase in the interior of a liquid half cylinder droplet of length L and radius *R* is:

$$2RL\Delta p = 2\Upsilon L \quad or \quad \Delta p = \frac{\Upsilon}{R}$$

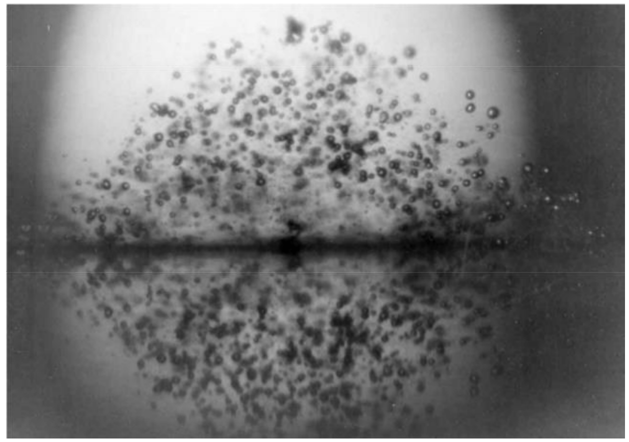
- Contact angle θ appears when a liquid interface intersects with a solid surface.

$$\theta = \begin{cases} < 90° & \textit{wetting liquid} \\ > 90° & \textit{nonwetting liquid} \end{cases}$$

- Water is extremely wetting to a clean glass surface with $\theta \approx 0$. For a clean mercury-air-glass interface, $\theta \approx 130°$.

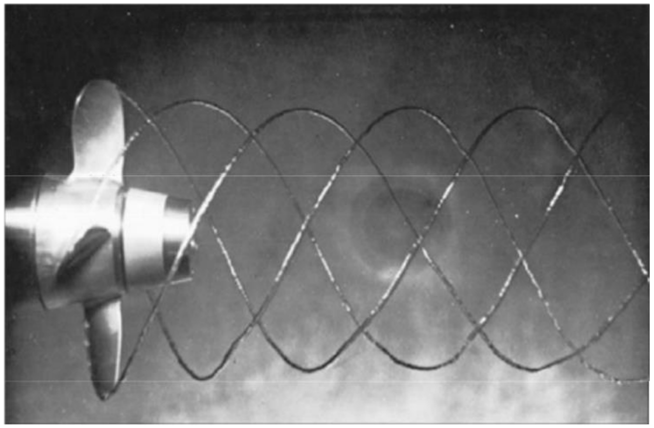
# Vapor pressure and cavitation

- Vapor pressure: the pressure at which a liquid boils and is in equilibrium with its own vapor.
- When the liquid pressure is dropped below the vapor pressure due to a flow phenomenon, we call the process *cavitation*.

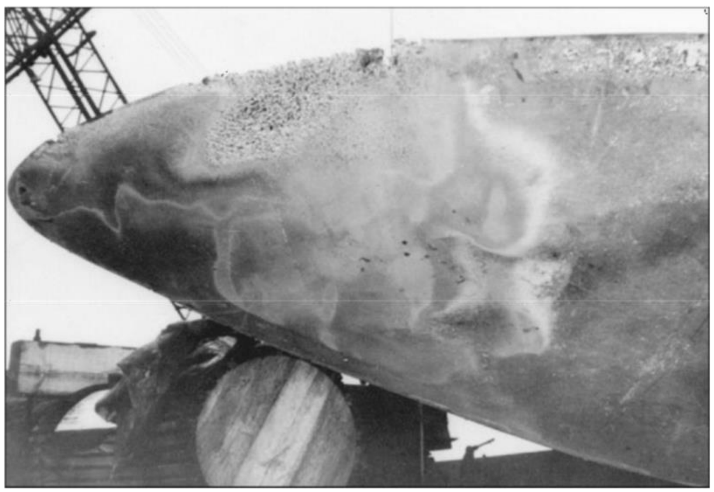


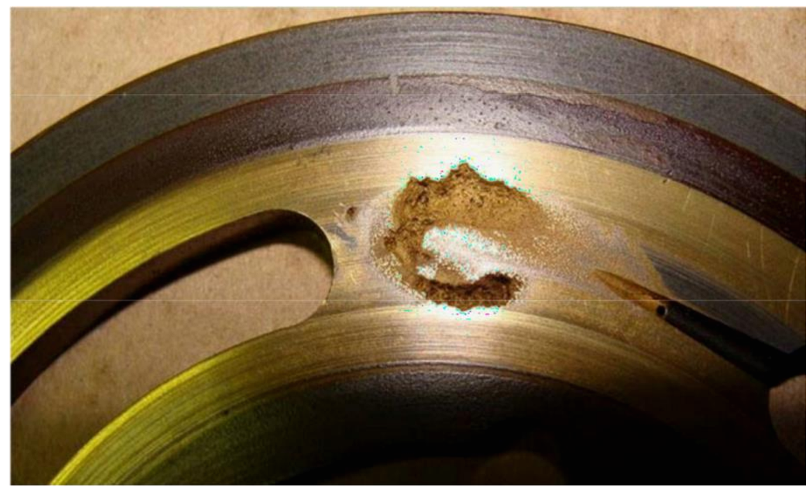
- The dimensionless parameter describing flow-induced boiling is called cavitation number:

$$Ca = \frac{p_a - p_v}{0.5\rho V^2}$$



- Bubble formation due to high velocity (flow-induced boiling).

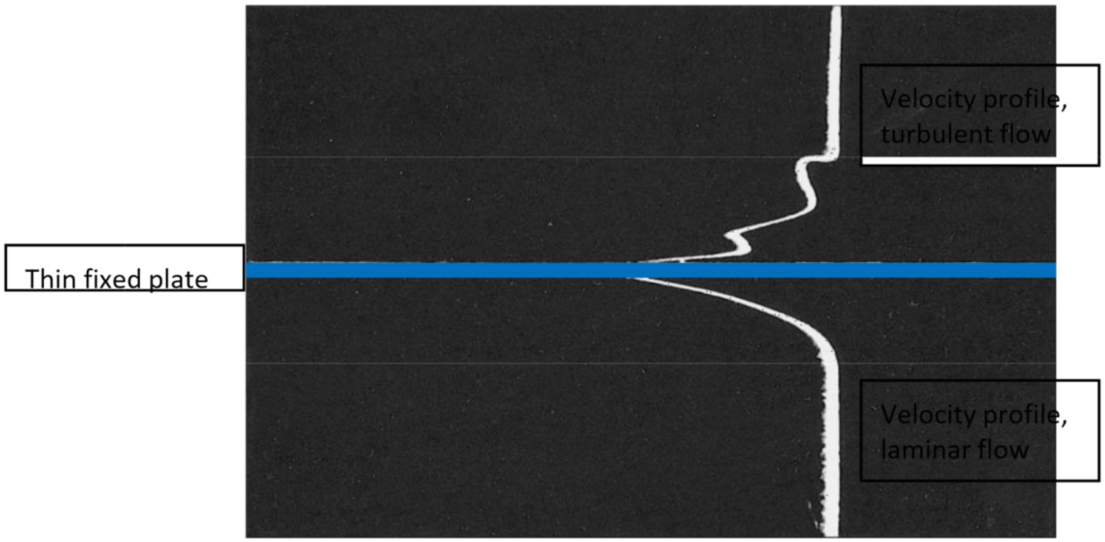




- Damage (erosion) due to cavitation on a marine propeller.

# No-slip and no-temp jump

- When a fluid flow is bounded by a surface, molecular interactions cause the fluid in contact with the surface to seek momentum and energy equilibrium with the surface.




$V_{fluid} = V_{wall}$ No-slip flow condition

$T_{fluid} = T_{wall}$ No-temperature jump condition

# Speed of sound & compressibility

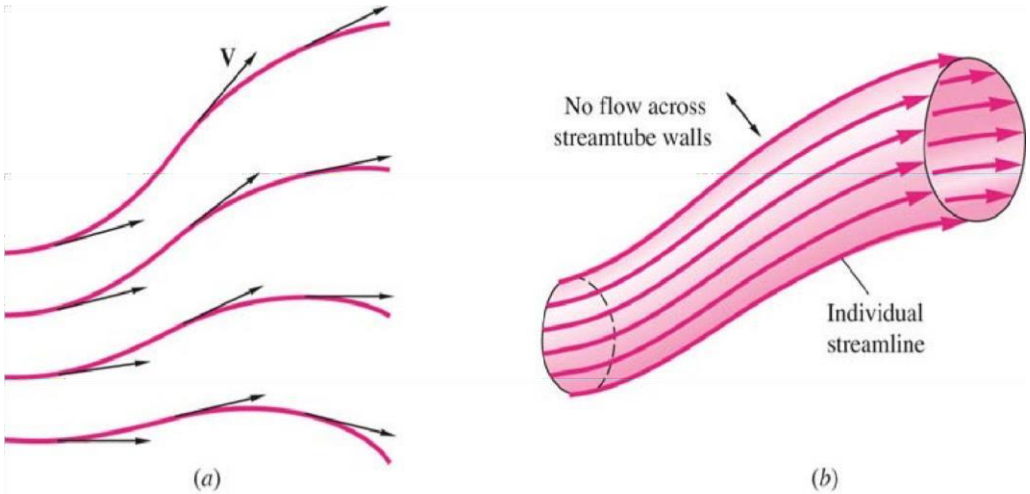
- the compressibility effects are important at high gas flows due to significant density changes. 

- Speed of sound: is the rate of propagation of small disturbance pressure pulses (sound waves) through the fluid:

$$a^2 = k\left(\frac{\partial p}{\partial \rho}\right)_T, \qquad k = \frac{c_p}{c_v}$$

- For an ideal gas $a_{ideal\ gas} = \sqrt{kRT}$
- Mach number is the ratio of the flow to the speed of sound $Ma = \frac{V}{a}$
- Compressibility effects are normally neglected for Ma<0.3

# Flow pattern




- Streamline: is a line everywhere tangent to the velocity vector at a given instant.
- Pathline: is the actual path traversed by a given fluid particle.
- **Note**: in steady flows, streamlines and pathlines are identical.

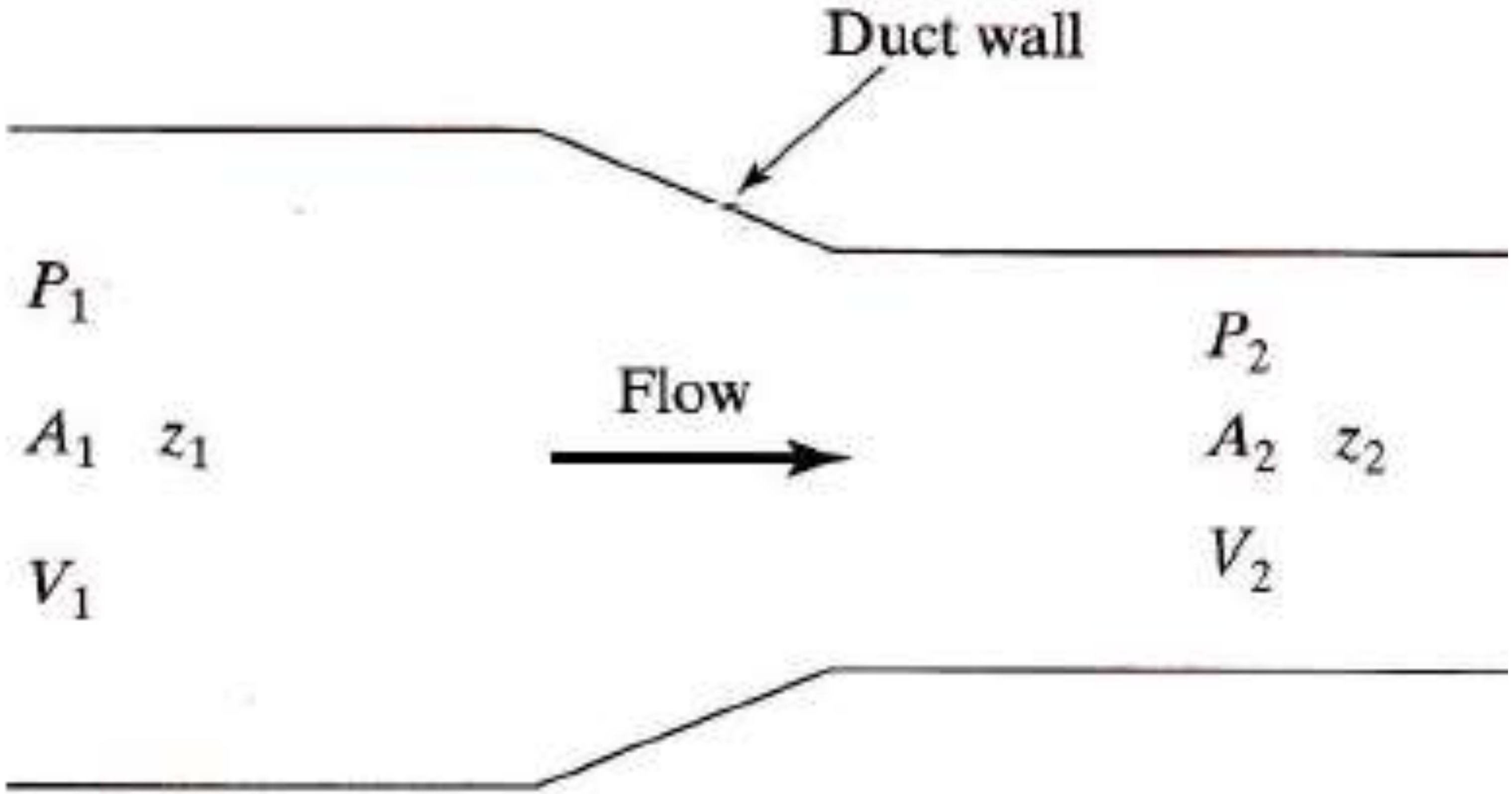
- If the elemental arc length *dr*of a streamline is to be parallel to *V*, their respective components must be in proportion:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V}$$

- Bernoulli equation takes the form of

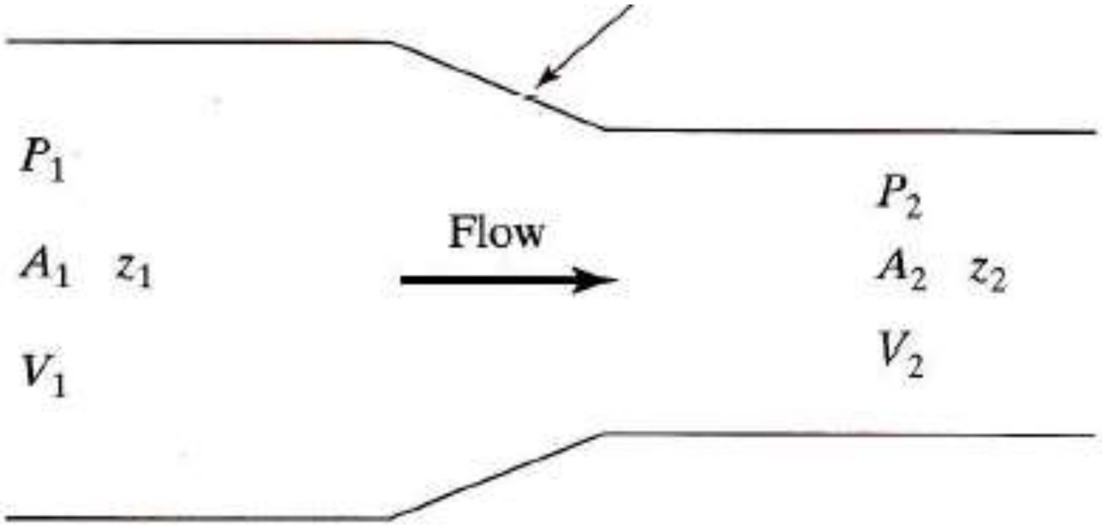
$$\frac{V_1^2}{2} + \frac{P_1}{\rho} + gz_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho} + gz_2$$

where V is the fluid velocity, P is the fluid pressure, z is the elevation of the location in the pipe relative to a specified reference elevation (datum), ρ is the fluid density, and g is gravity




The velocities at two axial locations in the duct with different areas are related through the conservation of mass equation,

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \dot{m}$$




where, A is the duct cross-sectional area and $\dot{m}$ is the fluid mass flow rate (e.g., kg/s).
For an incompressible fluid, the density is constant.

conservation of mass equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \dot{m}$$

$$\frac{V_1^2}{2} + \frac{P_1}{\rho} + gz_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho} + gz_2$$

*is usually written in the form:*

$$V_1 A_1 = V_2 A_2 = Q$$

where $Q$ is the volume flow rate (e.g., $m^3/s$).

Equations can be combined to obtain an expression

$$V_2 = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2[(P_1 + g\rho z_1) - (P_2 + g\rho z_2)]}{\rho}}$$

$$V_2 = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2[(P_1 + g\rho z_1) - (P_2 + g\rho z_2)]}{\rho}}$$

$$V_1 A_1 = V_2 A_2 = Q$$

$$Q = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2[(P_1 + g\rho z_1) - (P_2 + g\rho z_2)]}{\rho}}$$

***The <u>theoretical</u> basis for a class of flow meters in which the flow rate is determined from the pressure change caused by variation in the area of a conduit.***

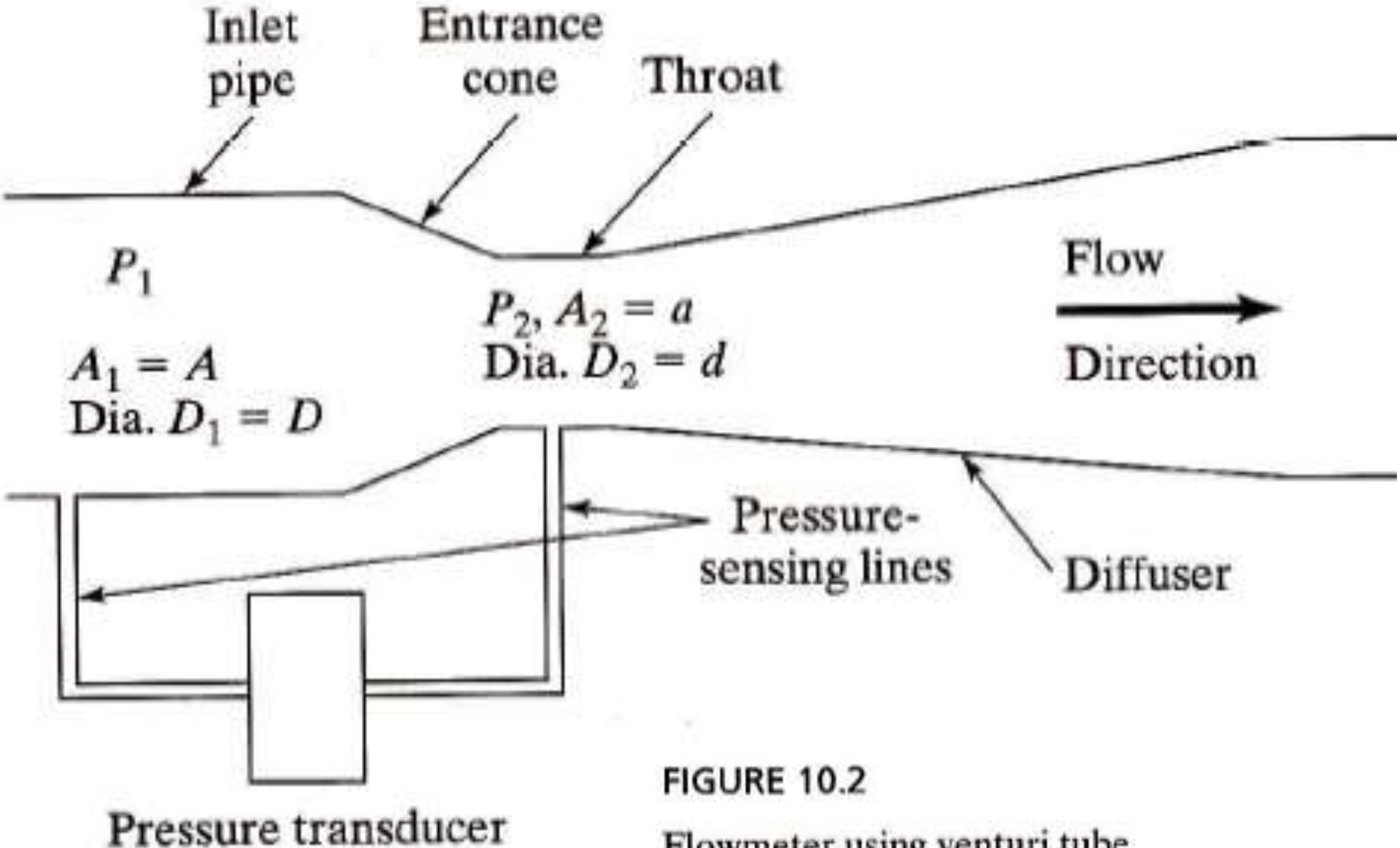
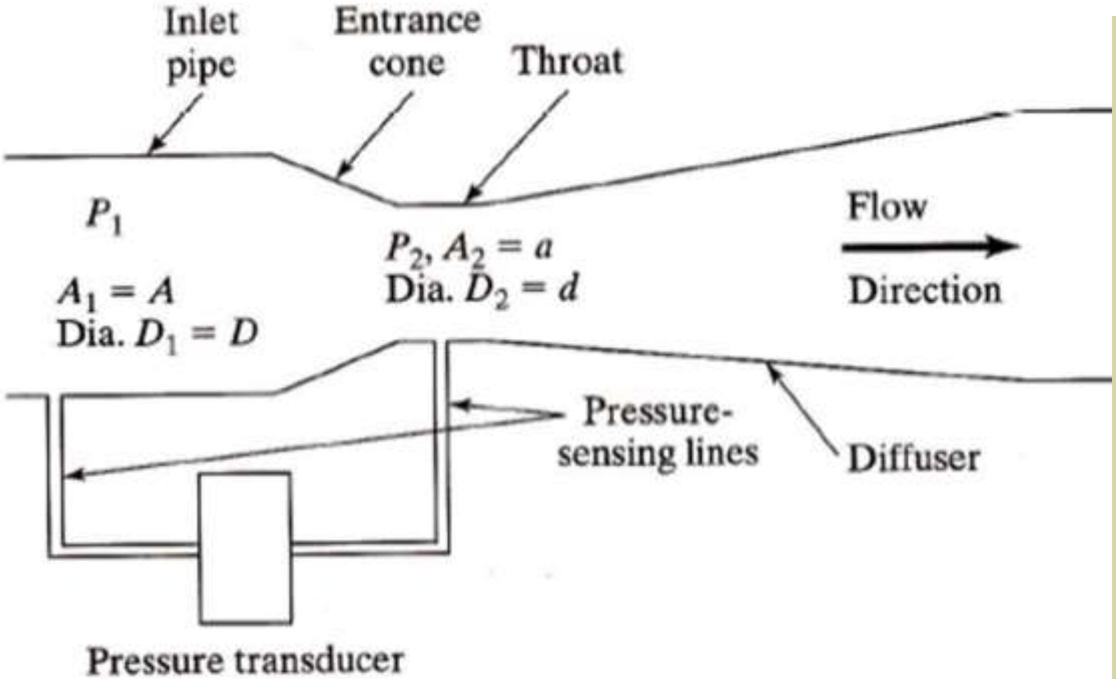
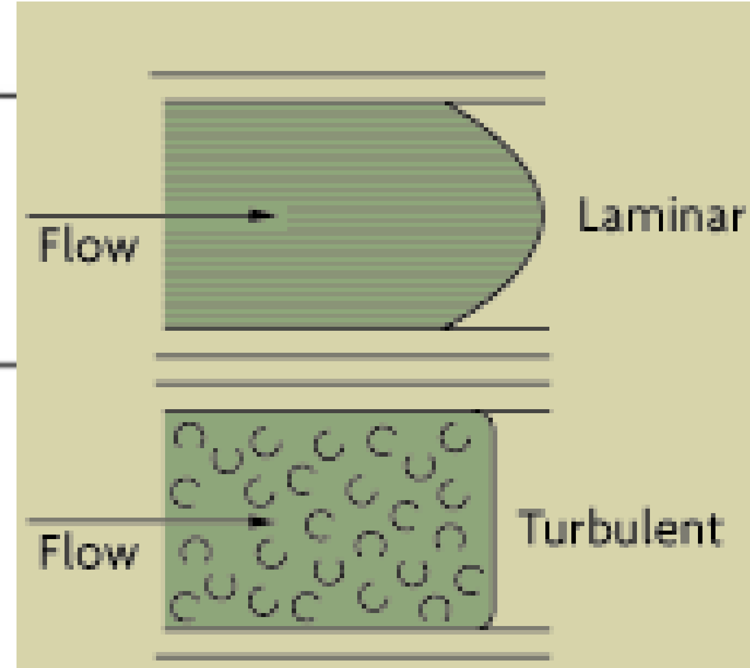



FIGURE 10.2

Flowmeter using venturi tube.




$$Q = \frac{C A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2[(P_1 + g\rho z_1) - (P_2 + g\rho z_2)]}{\rho}}$$

The factor $C$, called the *discharge coefficient*

is used to account for nonideal effects.

and a parameter called the Reynolds number, which is defined as

$$\text{Re} = \frac{\rho V D}{\mu}$$

$$Q = \frac{CA_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2[(P_1 + g\rho z_1) - (P_2 + g\rho z_2)]}{\rho}}$$

$$K = \frac{C}{\sqrt{1 - (A_2/A_1)^2}}$$

$K$, called the *flow coefficient*,

When $Z_1 = Z_2$ Flow Rate Equation becomes as follows:

$$Q = \frac{CA_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$