SAN JACINTO COLLEGE

STUDENT SUCCESS CENTER

Matrices & Matrix Addition, Subtraction, & Multiplication

According to Khan Academy, "a matrix is a **rectangular arrangement of numbers into rows and columns**", where each number in a position is called an "**element**".

The "size" of a matrix is written in the form of *rows* × *columns.*

Examples of Matrices

Note the number of rows and columns in each matrix

Size	Example Matrix
2 × 1	$\begin{bmatrix} 6 \\ 11 \end{bmatrix}$
2 × 3	$\begin{bmatrix} 6 & 7 & 10 \\ 11 & 15 & 5 \end{bmatrix}$
3 × 2	$\begin{bmatrix} 6 & 7 \\ 11 & 15 \\ 0 & 19 \end{bmatrix}$
3 × 4	$\begin{bmatrix} 25 & -1 & 6 & 7 \\ -79 & 29 & 11 & 15 \\ -17 & 19 & 0 & 19 \end{bmatrix}$
5 × 5 (a type of "square" matrix)	$\begin{bmatrix} 2 & 5 & 4 & 10 & 4 \\ 15 & 16 & 0 & 2 & 9 \\ 5 & 10 & 13 & 19 & 21 \\ 26 & 4 & 0 & 18 & 31 \\ 1 & 17 & 22 & 36 & 19 \end{bmatrix}$
1 × 6	$\begin{bmatrix} 2 & 6 & 14 & 10 & 4 & 19 \end{bmatrix}$
3 × 3 (a type of "square" matrix)	$\begin{bmatrix} 22 & 5 & 5 \\ 31 & 49 & 1 \\ 0 & 52 & 16 \end{bmatrix}$

Addition & Subtraction

$$A + B = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 4 \end{bmatrix}$$

In order to add or subtract matrices, the size of the matrices must be the **same**.

Examples

$$\underset{3 \times 2}{\begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 8 & 9 \end{bmatrix}} + \underset{2 \times 2}{\begin{bmatrix} 5 & 7 \\ 8 & 3 \end{bmatrix}} = \mathit{UNDEFINED}$$

Notice here how a 3×2 matrix is NOT the same as a 2×2 matrix. These two matrices CANNOT be added or subtracted.

$$\underset{3 \times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 8 & 9 & 2 \end{bmatrix}} + \underset{3 \times 3}{\begin{bmatrix} 5 & 7 & 1 \\ 8 & 3 & 9 \\ 1 & 4 & 6 \end{bmatrix}} = \underset{3 \times 3}{\begin{bmatrix} 6 & 9 & 4 \\ 12 & 4 & 14 \\ 9 & 13 & 8 \end{bmatrix}}$$

Notice how both of the matrices have the same size, 3×3. These two matrices CAN be added or subtracted.

$$\underset{1 \times 2}{\begin{bmatrix} 4 & 6 \end{bmatrix}} + \underset{2 \times 1}{\begin{bmatrix} 5 \\ 8 \end{bmatrix}} = \mathit{UNDEFINED}$$

Notice how the two matrices are NOT the same size. These two matrices CANNOT be added or subtracted.

Adding & Subtracting Matrices

$$\text{Adding} \rightarrow \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 8 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 11 & 10 \end{bmatrix}$$

$$\text{Subtracting} \rightarrow \begin{bmatrix} 5 & 9 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 7 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$$

Adding and subtracting matrices is very simple. Once you know the matrices are the same size, just add/subtract the **numbers in the position of the first matrix** with the **number in the same position of the other matrix**.

$$A + B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

Examples

$$\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} - \begin{bmatrix} 7 & 6 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 4-7 & 8-6 \\ 12-9 & 16-12 \end{bmatrix} \rightarrow \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 9 \end{bmatrix} + \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 3+5 \\ 9+7 \end{bmatrix} \rightarrow \begin{bmatrix} 8 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12 \end{bmatrix} - \begin{bmatrix} 8 & 11 \end{bmatrix} = \begin{bmatrix} 6-8 & 12-11 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 9 & -8 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ -7 & 9 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3+(-4) & 6+5 \\ 9+(-7) & (-8)+9 \\ (-3)+2 & 1+0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 11 \\ 2 & 1 \\ -1 & 1 \end{bmatrix}$$

Multiplication

$$A \times B = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ 43 \end{bmatrix}$$

Just like adding and subtracting, we first need to take a look at the size of the two matrices we want to multiply.

Matrix A	Matrix B
$rows \times columns$	$rows \times columns$

The **number of columns in the first matrix** MUST be the same as the **number of rows in the second matrix**, otherwise, the answer is "undefined".

The answer, or resultant matrix, will have the **same number of rows as the first matrix** and the **same number of columns as the second matrix**.

Examples

$$\underset{4 \times 3}{\begin{bmatrix} 8 & 6 & 5 \\ 9 & 2 & 3 \\ 1 & 5 & 4 \\ 7 & 1 & 2 \end{bmatrix}} \times \underset{3 \times 1}{\begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}} = \underset{4 \times 1}{\begin{bmatrix} 56 \\ 53 \\ 18 \\ 40 \end{bmatrix}}$$

Here, **3 = 3**, so the final matrix will be of size, **4×1**

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} \times \underset{2 \times 1}{\begin{bmatrix} 7 \\ 12 \end{bmatrix}} = \mathit{UNDEFINED}$$

Here, **3 ≠ 2**, so the answer is "**undefined**"

Multiplying Matrices

Once we’ve checked the **number of columns of the first matrix is the same as the number of rows in the second matrix**, we can now multiply them together, however, this is where it gets tricky.

Let’s use this as an example:

$$\underset{2 \times 3}{\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix}} \times \underset{3 \times 2}{\begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix}} = \underset{2 \times 2}{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}$$

Here, **3 = 3**, so the final matrix will be of size, **2×2**

1. Solving for the first element of the answer:

$$\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} = \begin{bmatrix} (8 \times -5)+(1 \times 0)+(2 \times 1) & \square \\ \square & \square \end{bmatrix} \rightarrow \begin{bmatrix} -62 & \square \\ \square & \square \end{bmatrix}$$

2. Solving for the second element of the answer:

$$\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} = \begin{bmatrix} -62 & (8 \times 1)+(1 \times 2)+(2 \times 7) \\ \square & \square \end{bmatrix} \rightarrow \begin{bmatrix} -62 & 24 \\ \square & \square \end{bmatrix}$$

3. Solving for the third element of the answer:

$$\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} = \begin{bmatrix} -62 & 24 \\ (-5 \times -5)+(6 \times 0)+(7 \times -11) & \square \end{bmatrix} \rightarrow \begin{bmatrix} -62 & 24 \\ -52 & \square \end{bmatrix}$$

4. Solving for the fourth element of the answer:

$$\begin{bmatrix} 8 & 1 & 2 \\ -5 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix} = \begin{bmatrix} -62 & 24 \\ -52 & (-5 \times 1)+(6 \times 2)+(7 \times 7) \end{bmatrix} \rightarrow \begin{bmatrix} -62 & 24 \\ -52 & 56 \end{bmatrix}$$

Example

$$\overset{4 \times 3}{\begin{bmatrix} -5 & 2 & 0 \\ 7 & -3 & 4 \\ -1 & 3 & 2 \\ -5 & -3 & 2 \end{bmatrix}} \times \overset{3 \times 2}{\begin{bmatrix} -5 & 1 \\ 0 & 2 \\ -11 & 7 \end{bmatrix}} = \overset{4 \times 2}{\begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}}$$

Here, **3 = 3**, so the final matrix will be of size, **4×2**

$$= \begin{bmatrix} (-5 \times -5)+(2 \times 0)+(0 \times -11) & (-5 \times 1)+(2 \times 2)+(0 \times 7) \\ (7 \times -5)+(-3 \times 0)+(4 \times -11) & (7 \times 1)+(-3 \times 2)+(4 \times 7) \\ (-1 \times -5)+(3 \times 0)+(2 \times -11) & (-1 \times 1)+(3 \times 2)+(2 \times 7) \\ (-5 \times -5)+(-3 \times 0)+(2 \times -11) & (-5 \times 1)+(-3 \times 2)+(2 \times 7) \end{bmatrix}$$

$$= \begin{bmatrix} 25+0+0 & (-5)+4+0 \\ (-35)+0+(-44) & 7+(-6)+28 \\ 5+0+(-22) & (-1)+6+14 \\ 25+0+-22 & (-5)+(-6)+14 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & -1 \\ -79 & 29 \\ -17 & 19 \\ 3 & 3 \end{bmatrix}$$

Works Cited

Estela, Mike. "Adding and Subtracting Matrices." *ChiliMath*, ChiliMath, 19 July 2020, www.chilimath.com/lessons/advanced-algebra/adding-subtracting-matrices/.

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Pierce, Rod. "How to Multiply Matrices", *Math Is Fun*, 23 Aug. 2020, www.mathsisfun.com/algebra/matrix-multiplying.html.