Electrical Engineering 1st semester

Subject: Principles of Electrical Engineering

CHAPTER-1
FUNDAMENTALS

CHARGE:-

- The most basic quantity in an electric circuit is the electric charge.
- Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). Charge, positive or negative, is denoted by the letter q or Q.
- All matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge 'e' on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron and the neutron has no charge. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

CURRENT:-

- Current can be defined as the motion of charge through a conducting material, measured in Ampere (A). Electric current, is denoted by the letter i or I.
- The unit of current is the ampere abbreviated as (A) and corresponds to the quantity of total charge that passes through an arbitrary cross section of a conducting material per unit second.

  Mathematically,

$$I = \frac{Q}{t} \quad \text{or} \quad Q = It \qquad \text{.........(1.1)}$$

  Where Q is the symbol of charge measured in Coulombs (C), I is the current in amperes (A) and t is the time in second (s).

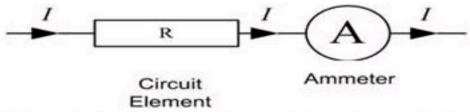
- The current can also be defined as the rate of charge passing through a point in an electric circuit. Mathematically,

$$i = \frac{dq}{dt} \qquad \text{............ (1.2)}$$

- The charge transferred between time t_1 and t_2 is obtained as

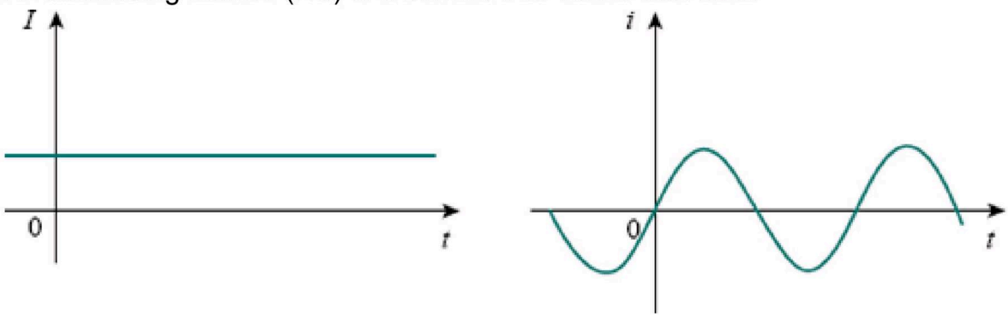
$$q = \int_{t1}^{t2} i\,dt \qquad \text{……….. (1.3)}$$

- A constant current (also known as a direct current or DC) is denoted by symbol I whereas a time-varying current (also known as alternating current or AC) is represented by the symbol i or i (t).
- Current is always measured through a circuit element in ammeter as shown in Fig.1.1



(**Fig. 1.1.** Current through Resistor (R))

- Two types of currents:
  1) A direct current (DC) is a current that remains constant with time.
  2) An alternating current (AC) is a current that varies with time.

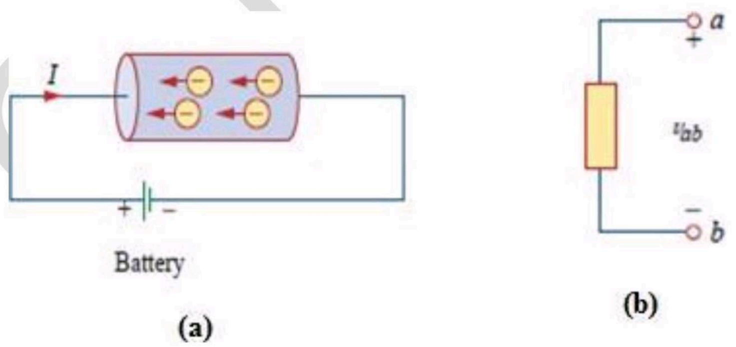


(a) (b)

(**Fig.1.2.** Two common types of current: (a) Direct Current (b) Alternating Current)

VOLTAGE (OR) POTENTIAL DIFFERENCE:-

- To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig 1.3(a). This emf is also known as voltage or potential difference. The voltage V_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b.



(**Fig. 1.3**.(a) Electric Current in a conductor, (b) Polarity of voltage V_{ab})

- Voltage (or potential difference) is the energy required to move charge from one point to the other, measured in volts (V). Voltage is denoted by the letter v or V.

  Mathematically,

$$V_{ab} = \frac{dw}{dt} \qquad \text{……….(1.4)}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage Vab or simply V is measured in volts (V).

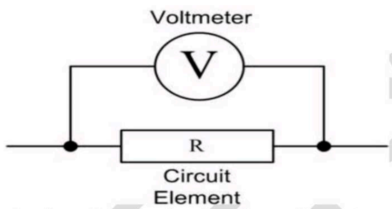
1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

Fig1.3(b). shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity.

- The V_{ab} can be interpreted in two ways: (1) point a is at a potential of V_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is V_{ab}. It follows logically that in general

$$V_{ab} = -V_{ba} \quad \text{.........(1.5)}$$

- Voltage is always measured across a circuit element in Voltmeter as shown in Fig.1.4



(**Fig.1.4.** Measurement of voltage through voltmeter across circuit element)

POWER:-

- Power is the time rate of expending or absorbing energy, measured in watts (W). Power, is denoted by the letter p or P.
  Mathematically,

$$P = \frac{dw}{dt} \quad \text{..........(1.6)}$$

Where P is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From voltage and current equations, it follows that;

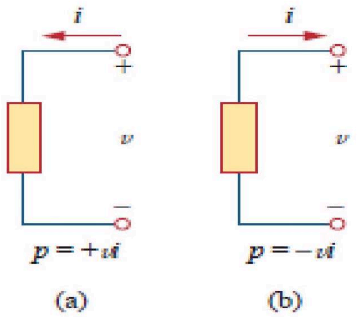
$$P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt} = V \times I \quad \text{..........(1.7)}$$

Thus, if the magnitude of current I and voltage are given, then power can be evaluated as the product of the two quantities and is measured in watts (W).

- **Sign of power:**

Plus sign: Power is absorbed by the element. (Resistor, Inductor)
Minus sign: Power is supplied by the element. (Battery, Generator)

- **Passive sign convention:**
  If the current enters through the positive polarity of the voltage, P = +VI
  If the current enters through the negative polarity of the voltage, P = – VI



(**Fig.1.5.** Polarities for Power using passive sign convention
(a) Absorbing Power (b) Supplying Power)

ENERGY:-

- Energy is the capacity to do work, and is measured in joules (J).
- The energy absorbed or supplied by an element from time 0 to t is given by,

$$W = \int_0^t P dt = \int_0^t VI dt \qquad \text{..........(1.8)}$$

- The electric power utility companies measure energy in watt-hours (WH) or Kilo watt-hours (KWH).

1 WH = 3600 J ……….(1.9)

ENERGY SOURCES:-

- The energy sources which are having the capacity of generating the energy. The most important energy sources are voltage or current sources that generally deliver power/energy to the circuit connected to them.

  There are two kinds of sources
  a) Independent sources
  b) Dependent sources

a) Independent Sources:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

Ideal Independent Voltage Source:

An ideal independent voltage source is an active element that gives a constant voltage across its terminals irrespective of the current drawn through its terminals.

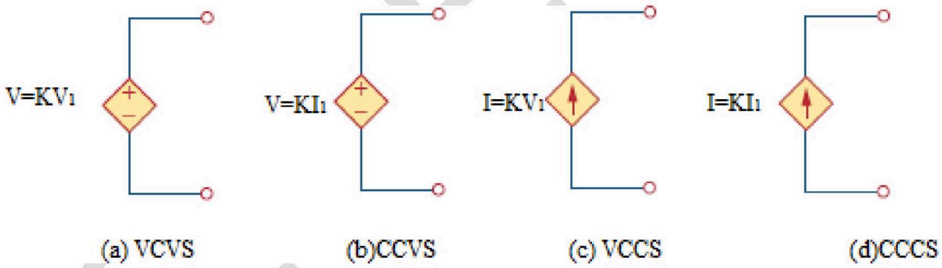
Ideal Independent Current Source:

An ideal independent Current source is an active element that gives a constant current through its terminals irrespective of the voltage appearing across its terminals.

b) Dependent (Controlled) Sources:

- An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.
- Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig.1.6 Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)



(**Fig.1.6.** (a) voltage-controlled voltage source (b) current-controlled voltage source (c) voltage-controlled current source (d) current-controlled current source)

ELECTRICAL LOAD:-

- The electrical load is a device that consumes electrical energy in the form of the current and transforms it into other forms like heat, light, work etc.
- The electrical load are (a) Resistive (b) Inductive (c) Capacitive
- **Resistive Load** – The resistive load obstructs the flow of electrical energy in the circuit and converts it into thermal energy.
  Ex- Lamp, Heater
- **Inductive Load-** The inductive load has a coil which stores magnetic energy when the current pass through it.
  Ex- Generator, motor, transformer
- **Capacitive Load-** The capacitive load include energy stored in materials and device.
  Ex- capacitor bank and synchronous condenser

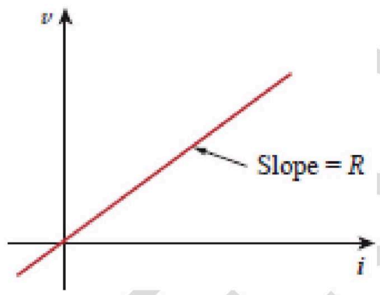
OHM'S LAW:-

- Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.
- Ohm's law states that at constant temperature, the voltage (V) across a conducting material is directly proportional to the current (I) flowing through the material. Mathematically,

$$V \propto I$$

$$V = RI \qquad \text{.........(1.10)}$$

Where the constant of proportionality R is called the resistance of the material. The V-I relation for resistor according to Ohm's law is depicted in Fig.1.7



(**Fig.1.7.** V-I Characteristics for resistor)

Limitations of Ohm's Law:

1. Ohm's law is not applicable to non-linear elements like diode, transistor etc.
2. Ohm's law is not applicable for non-metallic conductors like silicon carbide.

Example-1.1. An electrical iron carrying 2A at 120V. Find resistance of the device?

Solution:

$$R = \frac{V}{I} = \frac{120}{2} = 60\Omega$$

Example-1.2. The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 12Ω at 110V?

Solution:

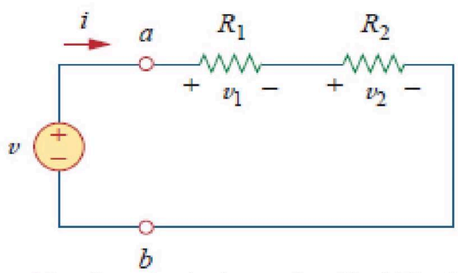
$$I = \frac{V}{R} = \frac{110}{12} = 9.167\,Amp$$

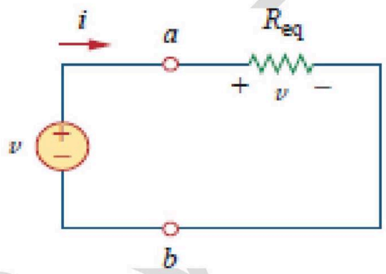
RESISTOR:-

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist the flow of current, is known as resistance and is represented by the symbol R. The Resistance is measured in ohms (Ω).

RELATION OF V,I &R IN SERIES CIRCUIT: -

Two or more resistors are said to be in series if the same current flows through all of them. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig.1.8





(a) (b)

(**Fig.1.8.** (a) A single loop circuit with two resistors in series, (b) Equivalent Circuit of series resistors)

The two resistors are in series, since the same current i flow in both of them. Applying Ohm's law to each of the resistors, we obtain

$$V_1 = iR_1, V_2 = iR_2 \qquad ....(1.11)$$

If we apply KVL in the loop (moving in the clockwise direction), we have

$$V - V_1 - V_2 = 0 \qquad ....(1.12)$$

Combining equation (1.11) &(1.12), we get

$$V = V_1 + V_2 = iR_1 + iR_2 = i(R_1 + R_2) \qquad ....(1.13)$$

Equation (1.13) can be written as $V = iR_{eq}$ ....(1.14)

Where $R_{eq} = R_1 + R_2$ i.e. the summation of two resistors.

From Eq.(1.13) we get,

$$i = \frac{V}{R_1 + R_2} \qquad ....(1.15)$$

In general, the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + ... + R_N = \sum_{n=1}^{N} R_n \qquad ....(1.16)$$

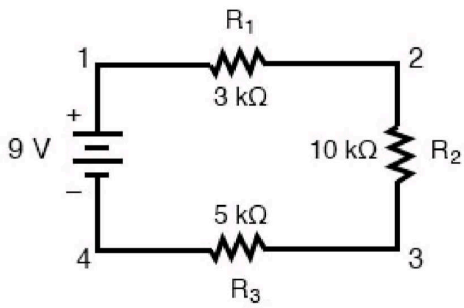
Voltage Division:

To determine the voltage across each resistor in Fig.1.8, we substitute Eq. (1.15) into Eq. (1.11) and obtain

$$V_1 = \frac{V}{R_1 + R_2} R_1, V_2 = \frac{V}{R_1 + R_2} R_2 \qquad ....(1.17)$$

Note that the source voltage is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division.

Example.1.3. Find the current I passing through and the voltage across each of the resistors in the circuit.



Solution: $R_{total} = R_1 + R_2 + R_3 = 3K\Omega + 10K\Omega + 5K\Omega = 18K\Omega$

$$I = \frac{V}{R_{total}} = \frac{9}{18 \times 10^3} = 0.5 \text{ mA}$$

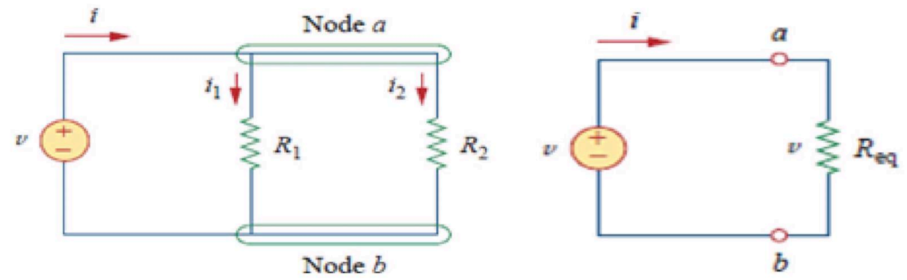
$$V_{R1} = \frac{VR1}{R_1 + R_2 + R_3} = \frac{9}{18 \times 10^3} \times 3 \times 10^3 = 1.5V$$

$$V_{R2} = \frac{V}{R_1 + R_2 + R_3} R_2 = \frac{9}{18 \times 10^3} \times 10 \times 10^3 = 5V$$

$$V_{R3} = \frac{V}{R_1 + R_2 + R_3} R3 = = 2.5V$$

RELATION OF V,I &R IN PARALLEL CIRCUIT:-

Two or more resistors are said to be in parallel if the same voltage appears across each element. Consider the circuit in Fig.1.9(a) , where two resistors are connected in parallel and therefore have the same voltage across them.



(a) (b)

(**Fig: 1.9.** (a) Two resistors in parallel (b) Equivalent circuit)

$$v = i_1R_1 = i_2R_2 \qquad \text{..... (1.18)}$$

$$i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2} \qquad \text{...... (1.19)}$$

Applying KCL at node a gives the total current i is

$$i = i_1 + i_2 \qquad \text{.....(1.20)}$$

Substituting Equation 1.19 into 1.20, we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \qquad \text{......(1.21)}$$

Where R_{eq} is the equivalent resistance of the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{......(1.22)}$$

Thus, The equivalent Resistance of parallel-connected resistors is the reciprocal of the sum of the reciprocals of the individual resistances.

If a circuit with N resistors in parallel then the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n} \qquad \text{.........(1.23)}$$

DIVISION OF CURRENT IN PARALLEL CIRCUIT:-

We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \quad ......(1.24)$$

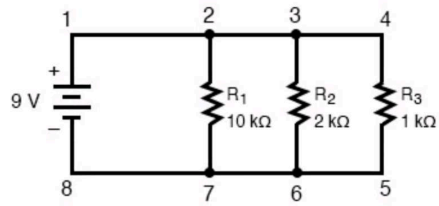
Substituting eq (1.24) into (1.19)

$$i_1 = \frac{iR_2}{R_1 + R_2}$$

$$i_2 = \frac{iR_1}{R_1 + R_2} \quad ....(1.25)$$

This shows that the total current is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit is known as a current divider.

Example.1.4. Find the current I passing through and the current passing through each of the resistors in the circuit below.



Solution:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10\times10^3} + \frac{1}{2\times10^3} + \frac{1}{1\times10^3} = 0.0016$$

$$R_{total} = 625\Omega$$

$$I = \frac{V}{R_{total}} = \frac{9}{625} = 0.0144Amp = 14.4mA$$

$$I_{R1} = \frac{V}{R_1} = \frac{9}{10\times10^3} = 0.9mA$$

$$I_{R2} = \frac{V}{R_2} = \frac{9}{2\times10^3} = 4.5mA$$

$$I_{R3} = \frac{V}{R_3} = \frac{9}{1\times10^3} = 9mA$$

POWER IN SERIES & PARALLEL CIRCUIT:-

(a) Series Combinations:- If the electrical appliances of power P_1& P_2 are connected in series with main voltage V having resistance R_1& R_2, then

Chapter-2

DC circuits

Electrical circuit : The close path flows by the electric circuit is known as electrical circuit.

DC circuits : The close path which DC current is flow is known as DC circuits.

Ohm's law : The current flowing between the end of the conductor is directly proportional to the potential difference across the end of the conductor with the physical condition, temp. pressure etc. don't change.

Mathematically,

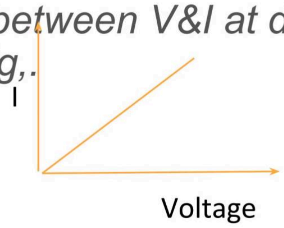
$$I \propto V$$

or $$V / I = \text{constant}$$

This constant is called the resistance of the conductor.

$$V / I = R$$

The ohm's law is verified , if the graph between V&I at different values is a straight line as shown in fig,.



• *Limitations of Ohm’s Law :*

I. It does not apply on the unilateral networks.

II. It does not apply on the non-linear networks , the parameter of the network is vary with the voltage and current. Their parameters like resistance, inductance, capacitance and frequency etc. do not remain constant with time. So, ohm’s law is not applicable to the non-linear networks.

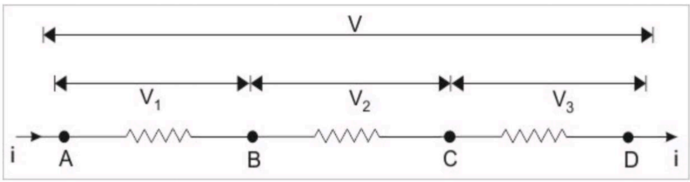
● *Applications of Ohm’ s Law:*

1. It can be applied to D.C as well as A. C circuit.
2. To find out the value resistance of the circuit and also for knowing the voltage and current of the circuit.

Resistance in series:

The circuit in which resistance are connected end to end so that they connected from one path for the flow of current than resistance are called connected in series and such circuit is called series circuit .

As shown in fig below resistance between point A and D is equal to the sum of three individual resistances. The current enters in to the point A of the combination, will also leave from point D as there is no other parallel path provided in the circuit. Now say this current is I. So this current I will pass through the resistance R_1, R_2 and R_3.



voltage drop across R1 , V1 = IR1

R2 , V2= IR2

R3 , V3= IR3

Since, sum of voltage drops across the individual resistance is nothing but the equal to applied voltage across the combination

Total voltage V = V1+V2+V3

V = IR1+IR2+IR3

V/ I = R1+R2+R3 then according to Ohm's law, V = IR

R = R1+R2+R3

So, the above proof shows that equivalent resistance of a combination of resistances in series is equal to the sum of individual resistance. If there were n number of resistances instead of three resistances, the equivalent resistance will be

$$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots R_n$$

Resistances in Parallel:

The circuit in which one end of each resistor is collected to common point and the other end of each resistor is connected to another common point so that they are many path for current flow then resistor are said to be connected in parallel and such circuit is called parallel circuit

As this current will get three parallel paths through these three electrical resistances, the current will be divided into three parts. Say currents I_1, I_1 and I_1 pass through resistor R_1, R_2 and R_3 respectively.

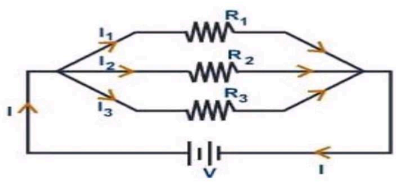
Acc to ohm's law

Current in resistance $R1$, $I1 = V / R1$

$R2$, $I2 = V / R2$

$R3$, $I3 = V / R3$

Total current $I = I1 + I2 + I3$



$I = V / R1 + V / R2 + V / R3$

$I = V[1 / R1 + 1 / R2 + 1 / R3$

$1 / R = 1 / R1 + 1 / R2 + 1 / R3$

The above expression represents equivalent resistance of resistor in parallel. If there were n number of resistances connected in parallel, instead of three resistances, the expression of equivalent resistance would be written as

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots\cdots+\frac{1}{R_n}\right)^{-1}$$

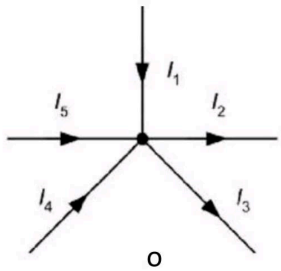
Hence, the number of resistor are connected in parallel reciprocal of total resistance is equal to the reciprocal sum of individual resistors.

i. *Voltage across each resistances of the parallel combinations is same.*
ii. *Current in each branch is given by the ohm law and total current is equal to the sum of branch current.*
iii. *Different resistance have individual current.*
iv. *Reciprocal of the total resistance is equal to the same of the reciprocal sum of the individual resistors.*

Application of parallel circuit : *In the domestic installation all the electrical appliances are connected in parallel across the supply so that voltage across each appliance is same . The reason for connecting the appliances in parallel are due to*

i. *Electrical appliances are rated for same voltage and operate efficiently when supplied with this rated voltage.*
ii. *When appliances are connected in parallel each operate independently of the others. therefore failure of an appliances will not effect the working of the others.*

Kirchhoff's current law(KCL) : The algebraic sum of all the current meeting at any junction in an electric circuit is zero. This is called the Kirchhoff's current law . If we take the sign of current following towards point O is taken as +ve and when current following away from the point O is taken as the –ve sign.



$I_1 - I_2 - I_3 + I_4 + I_5 = 0$

$+ I1 + I2 + (- I3) + I4 + (- I5) = 0$

$I1 + I2 + I4 = I3 + I5$

Incoming current = Outgoing current

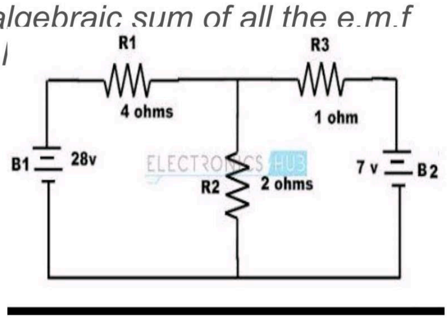
$$\sum_{n=1}^{N} i_n = 0$$

Where N is the total number of branches connected to a node.

$$\sum_{\text{node}} i_{enter} = \sum_{\text{node}} i_{leave}$$

Kirchhoff's voltage law (KVL) *: In any closed circuit the algebraic sum of the product of current and resistance(voltage drop) plus the algebraic sum of all the e.m.f in that circuit is equal to the zero is called Kirchhoff's voltage l*

i. ***Sign of e.m.f** : A rise in a potential is considered as +ve while fall in potential is considered as negative.*
ii. ***Sign of voltage drop** : There is voltage drop in resistance due to the flow of current through it . If we go with the current then voltage drop should be taken as –ve where as ,if we go against the current flow, then voltage drop should be taken as positive.*



Consider closed circuit ABCFA

Voltage drop in $R_1 = -(I_1+I_2)R_1$

By applying KVL to this loop

$-(I_1+I_2)R_1 + E_1 = 0$

Or $E_1 = (I_1+I_2)R_1$

Now consider loop CDEFC

Voltage drop in $R_2 = +I_2R_2$

By applying KVL to this loop

$I_2R_2 + (I_1+I_2)R_1 - E_2 = 0$

Or $E_2 = I_2R_2 + (I_1+I_2)R_1$

Thevenin's Theorem:

- Thevenin's theorem simplifies the process of solving for the unknown values of voltage and current in a network by reducing the network to an equivalent series circuit connected to any pair of network terminals.
- Any network with two open terminals can be replaced by a **single voltage source (V_{TH})** and a **series resistance (R_{TH})** connected to the open terminals. A component can be removed to produce the open terminals.

Vth = open circuit voltage between two terminals (known's as the Thevenin's equivalent voltage source. This is obtained by removing load resistance (RL)and find out the potential difference across the open terminal of RL. V_{TH} is determined by calculating the voltage between open terminals A and B.

Rth =It is the Thevenin's equivalent resistance which can be obtained by shorting the voltage source and calculating the circuit's total resistance as seen from open terminals A and B

Thevenin's Theorem:

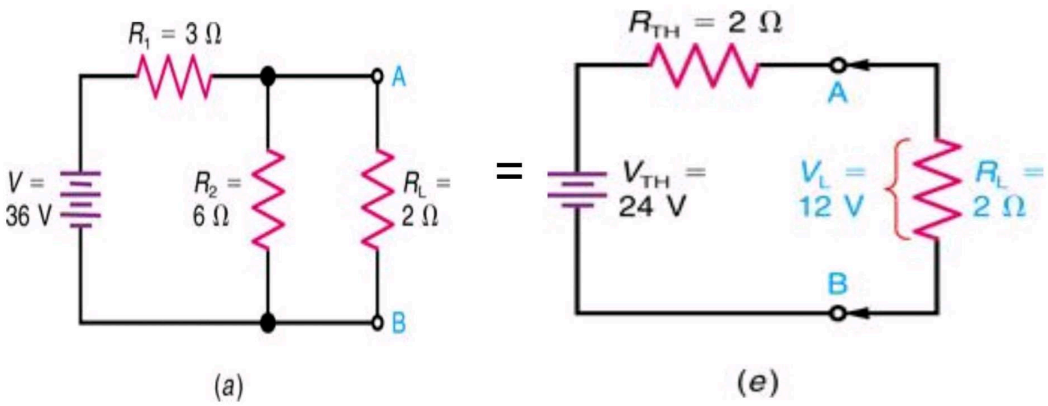


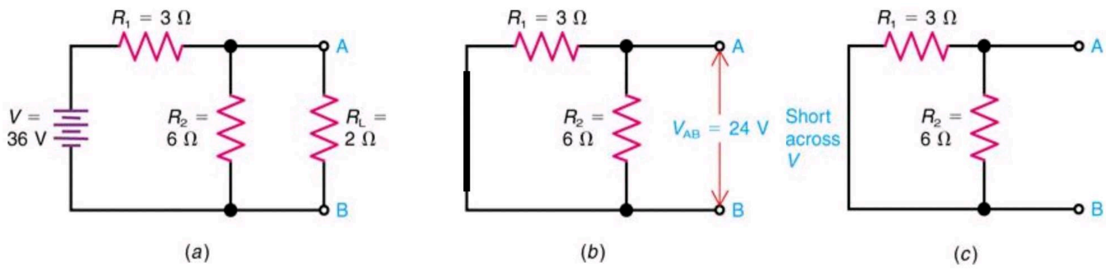
Fig. 1 Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin's Theorem

$$V_{R2} := 36V \cdot \frac{6\Omega}{3\Omega + 6\Omega} \qquad V_{R2} = 24\,V \qquad V_{AB} := V_{R2}$$

Fig. 2 Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit V to find that R_{AB} is 2Ω.

Thevenin's Theorem



$$R_{TH} := \frac{3\Omega \cdot 6\Omega}{3\Omega + 6\Omega} \qquad R_{TH} = 2\Omega$$

Fig. 3: Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin’s Theorem

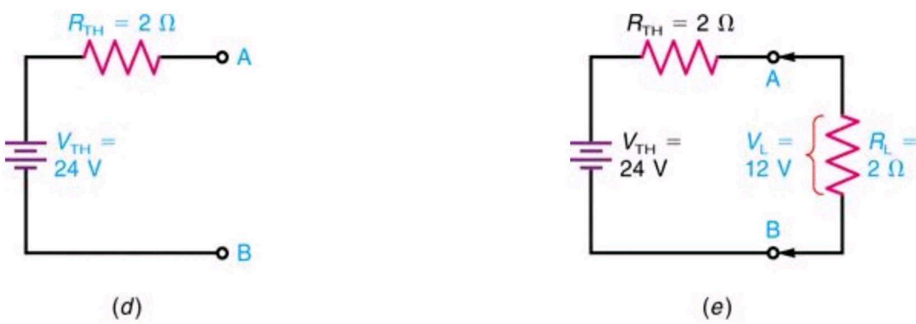
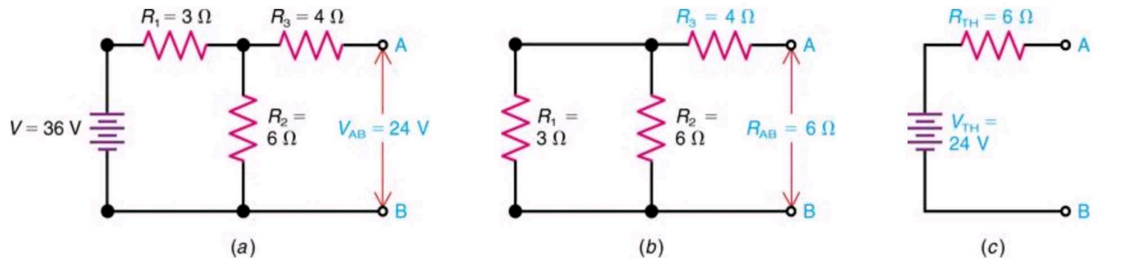


Fig.4 (*d*) Thevenin equivalent circuit. (*e*) Reconnect R_L at terminals A and B to find that V_L is 12V.

Thevenin's Theorem



Note that $\mathbf{R_3}$ does not change the value of V_{AB} produced by the source V, but R_3 <u>does</u> increase the value of R_{TH}.

$$R_{TH} := \frac{3\Omega \cdot 6\Omega}{3\Omega + 6\Omega} + 4\Omega \qquad R_{TH} = 6\Omega$$

Fig. :5 Thevenizing the circuit of Fig. 3 but with a 4-Ω R_3 in series with the A terminal. (*a*) V_{AB} is still 24V. (*b*) Now the R_{AB} is 2 + 4 = 6 Ω. (*c*) Thevenin equivalent circuit.

Norton’s Theorem:

- Norton’s theorem is used to simplify a network in terms of currents instead of voltages.
- It reduces a network to a simple parallel circuit with a current source (comparable to a voltage source).
- Norton’s theorem states that any network with two terminals can be replaced by a single current source and parallel resistance connected across the terminals.

Thevenin-Norton Conversions:

- **Thevenin’s theorem** says that any network can be represented by a voltage source and series resistance.
- **Norton’s theorem** says that the same network can be represented by a current source and shunt resistance.
- Therefore, it is possible to convert directly from a Thevenin form to a a Norton form and vice versa.
- Thevenin-Norton conversions are often useful.

Maximum Power Transfer

- For any power source, the maximum power transferred from the power source to the load is when the resistance of the load R_L is equal to the equivalent or input resistance of the power source ($R_{in} = R_{Th}$ or R_N).
  - The process used to make $R_L = R_{in}$ is called impedance matching.

Find the value of R_{load} that maximizes power

$$\frac{dp}{dR_L} = V_{Th}^2 \left(\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right) = 0$$

$$(R_{Th} + R_L)^2 = 2R_{load}(R_{Th} + R_L)$$

$$R_L = R_{Th}$$

The maximum power delivered to the load

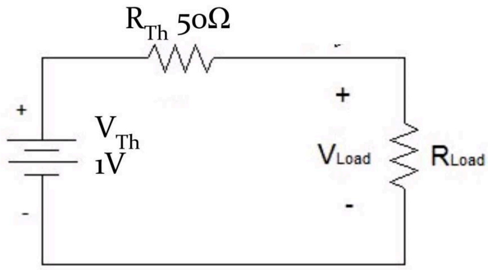
$$p_{max} = I^2 R_L = \frac{V_{Th}^2}{(2R_L)^2} R_L$$

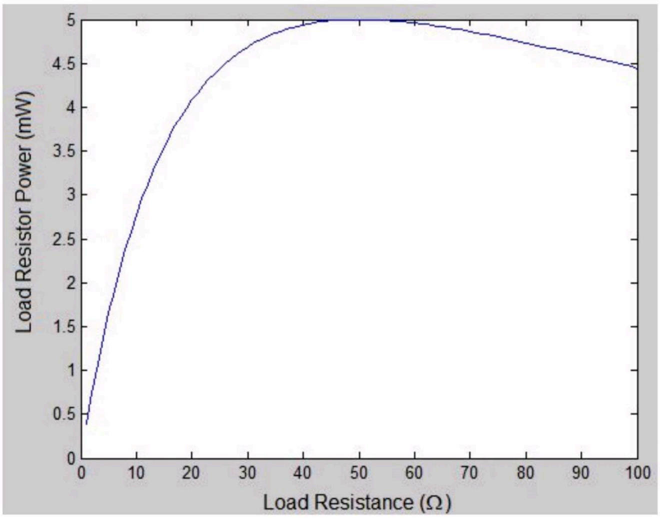
$$p_{max} = \frac{V_{Th}^2}{4R_L}$$

Power Transfer Calculation

$$P_L = V_L^{\,2} / R_L$$

$$P_L = \frac{\left[\frac{R_L}{R_L + R_{Th}} V_{Th}\right]^2}{R_L}$$

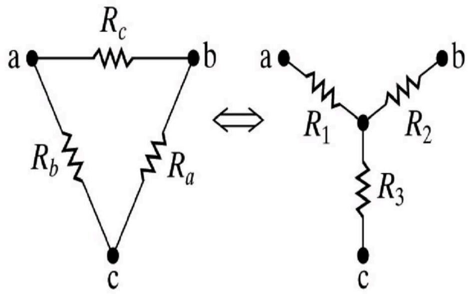




Application

- When developing new circuits for a known application, optimize the power transfer by designing the circuit to have an input resistance close to the load resistance.
- When selecting a source to power a circuit, one of the selection criteria is to match the input impedance to the load resistance.

Δ – Y Conversion



$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

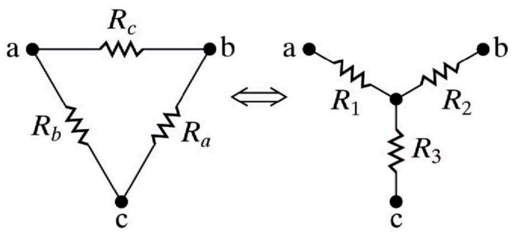
$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3$$

- The resistance between the terminal pairs must be the same for both circuits

Y – Δ Conversion

- After some algebraic manipulation



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

CHAPTER 3

Fundamentals of Electrostatics

- ***Electrostatics*** is the branch of electromagnetics dealing with the effects of electric charges at rest.
- The fundamental law of *electrostatics* is ***Coulomb's law***.

Electric Charge

- Electrical phenomena caused by friction are part of our everyday lives, and can be understood in terms of ***electrical charge***.
- Electrical charge is that entity due to presence of which a stationary particle can response in an electrostatic field.
- The effects of ***electrical charge*** can be observed in the attraction/repulsion of various objects when "charged."
- Charge comes in two varieties called "positive" and "negative."

Electric Charge

- Objects carrying a net positive charge attract those carrying a net negative charge and repel those carrying a net positive charge.
- Objects carrying a net negative charge attract those carrying a net positive charge and repel those carrying a net negative charge.
- On an atomic scale, electrons are negatively charged and nuclei are positively charged.

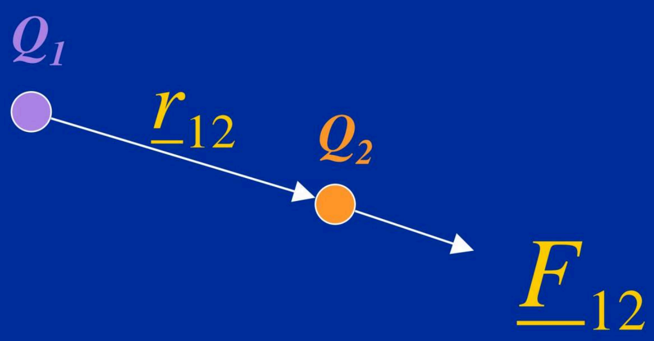
Electric Charge

- Electric charge is inherently quantized such that the charge on any object is an integer multiple of the smallest unit of charge which is the magnitude of the electron charge
  $e = 1.602 \times 10^{-19}$ C.
- On the macroscopic level, we can assume that charge is "continuous."

Coulomb's Law

- ***Coulomb's law*** is the "law of action" between charged bodies.
- ***Coulomb's law*** gives the electric force between two ***point charges*** in an otherwise empty universe.
- A ***point charge*** is a charge that occupies a region of space which is negligibly small compared to the distance between the point charge and any other object.

Coulomb's Law



Unit vector in direction of R_{12}

Force due to Q_1 acting on Q_2

$$\underline{F}_{12} = \hat{a}_{R_{12}} \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2}$$

Coulomb's Law

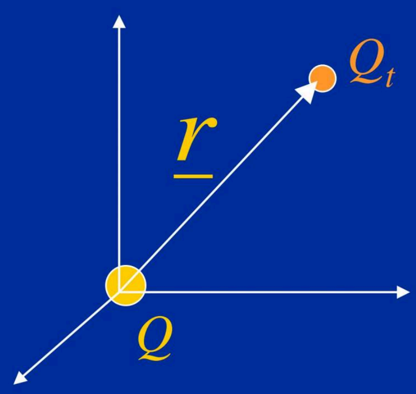
- The force on Q_1 due to Q_2 is equal in magnitude but opposite in direction to the force on Q_2 due to Q_1.

$$\overline{F}_{21} = -\overline{F}_{12}$$

Electric Field

- Consider a point charge Q placed at the <u>origin</u> of a coordinate system in an otherwise empty universe.
- A test charge Q_t brought near Q experiences a force:

$$\underline{F}_{Q_t} = \hat{a}_r \frac{QQ_t}{4\pi\varepsilon_0 r^2}$$



Electric Field

- The existence of the force on Q_t can be attributed to an ***electric field*** produced by Q.
- The ***electric field*** produced by Q at a point in space can be defined as the force per unit charge acting on a test charge Q_t placed at that point.

$$\overline{E} = \lim_{Q_t \to 0} \frac{\overline{F}_{Q_t}}{Q_t}$$

Electric Field

- The basic units of electric field are *newtons per coulomb*.
- In practice, we usually use *volts per meter*.

Continuous Distributions of Charge

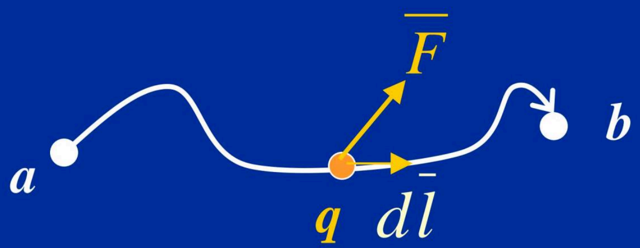
- Charge can occur as
  - *point charges* (C)
  - *volume charges* (C/m^3) ⇐ **most general**
  - *surface charges* (C/m^2)
  - *line charges* (C/m)

Electrostatic Potential

- An electric field is a *force field*.
- If a body being acted on by a force is moved from one point to another, then ***work*** is done.
- The concept of ***scalar electric potential*** provides a measure of the work done in moving charged bodies in an electrostatic field.

Electrostatic Potential

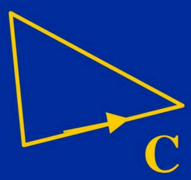
- The work done in moving a test charge from one point to another in a region of electric field:



$$W_{a\to b} = -\int_a^b \underline{F}\cdot d\underline{l} = -q\int_a^b \underline{E}\cdot d\underline{l}$$

Electrostatic Potential

- The electrostatic field is ***conservative***:
  - The value of the line integral depends only on the end points and is independent of the path taken.
  - The value of the line integral around any closed path is zero.



$$\oint_C \underline{E} \cdot d\underline{l} = 0$$

Electrostatic Potential

- The work done per unit charge in moving a test charge from point *a* to point *b* is the ***electrostatic potential difference*** between the two points:

$$V_{ab} \equiv \frac{W_{a\to b}}{q} = -\int_a^b \underline{E} \cdot d\underline{l}$$

electrostatic potential difference
Units are volts.

Electrostatic Potential

- Since the electrostatic field is conservative we can write

$$V_{ab} = -\int_a^b \underline{E} \bullet d\underline{l} = -\int_a^{P_0} \underline{E} \bullet d\underline{l} - \int_{P_0}^b \underline{E} \bullet d\underline{l}$$

$$= -\int_{P_0}^b \underline{E} \bullet d\underline{l} - \left(-\int_{P_0}^a \underline{E} \bullet d\underline{l} \right)$$

$$= V(b) - V(a)$$

Electrostatic Potential

- Thus the ***electrostatic potential*** V is a scalar field that is defined at every point in space.
- In particular the value of the *electrostatic potential* at any point P is given by

$$V(\underline{r}) = -\int_{P_0}^{P} \underline{E} \bullet d\underline{l}$$

P_0 ← **reference point**

Electrostatic Potential

- The ***reference point*** (P_0) is where the potential is zero (analogous to ***ground*** in a circuit).
- Often the reference is taken to be at infinity so that the potential of a point in space is defined as

$$V(\underline{r}) = -\int_{\infty}^{P} \underline{E} \bullet d\underline{l}$$

Electrostatic Potential and Electric Field

- The work done in moving a point charge from point *a* to point *b* can be written as

$$W_{a\to b} = QV_{ab} = Q\{V(b) - V(a)\}$$

$$= -Q\int_a^b \underline{E} \bullet d\underline{l}$$

Electrostatic Potential and Electric Field

- Along a short path of length Δl we have

$$\Delta W = Q\Delta V = -Q\underline{E} \cdot \Delta \underline{l}$$

or

$$\Delta V = -\underline{E} \cdot \Delta \underline{l}$$

Electrostatic Potential and Electric Field

- Along an incremental path of length *dl* we have

$$dV = -\underline{E} \cdot d\underline{l}$$

- Recall from the definition of *directional derivative*:

$$dV = \nabla V \cdot d\underline{l}$$

Electrostatic Potential and Electric Field

- Thus:

$$\underline{E} = -\nabla V$$

the "del" or "nabla" operator

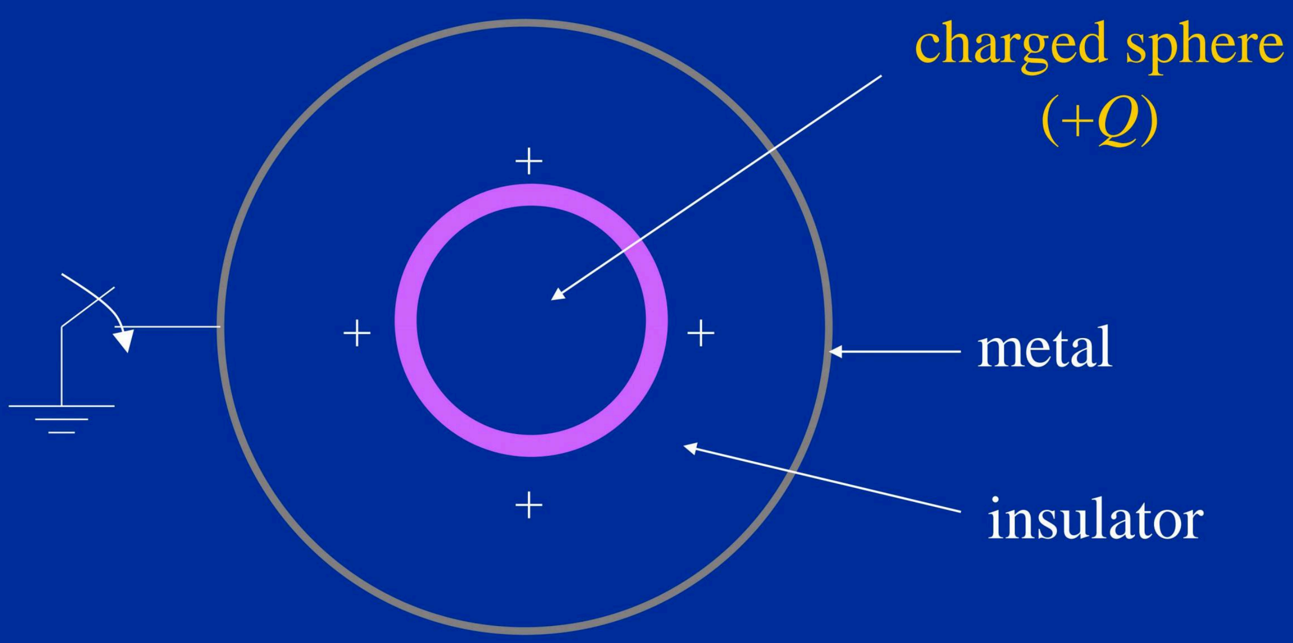
Visualization of Electric Fields

- An electric field (like any vector field) can be visualized using ***flux lines*** (also called ***streamlines*** or ***lines of force***).
- A ***flux line*** is drawn such that it is everywhere tangent to the electric field.
- A ***quiver plot*** is a plot of the field lines constructed by making a grid of points. An arrow whose tail is connected to the point indicates the direction and magnitude of the field at that point.

Visualization of Electric Potentials

- The scalar electric potential can be visualized using ***equipotential surfaces***.
- An ***equipotential surface*** is a surface over which V is a constant.
- Because the electric field is the negative of the gradient of the electric scalar potential, the electric field lines are everywhere normal to the equipotential surfaces and point in the direction of decreasing potential.

Faraday's Experiment



Faraday's Experiment (Cont'd)

- Two concentric conducting spheres are separated by an insulating material.
- The inner sphere is charged to $+Q$. The outer sphere is initially uncharged.
- The outer sphere is *grounded* momentarily.
- The charge on the outer sphere is found to be $-Q$.

Faraday's Experiment (Cont'd)

- Faraday concluded there was a "*displacement*" from the charge on the inner sphere through the inner sphere through the insulator to the outer sphere.
- The ***electric displacement*** (or ***electric flux***) is equal in magnitude to the charge that produces it, independent of the insulating material and the size of the spheres.

CHAPTER 4

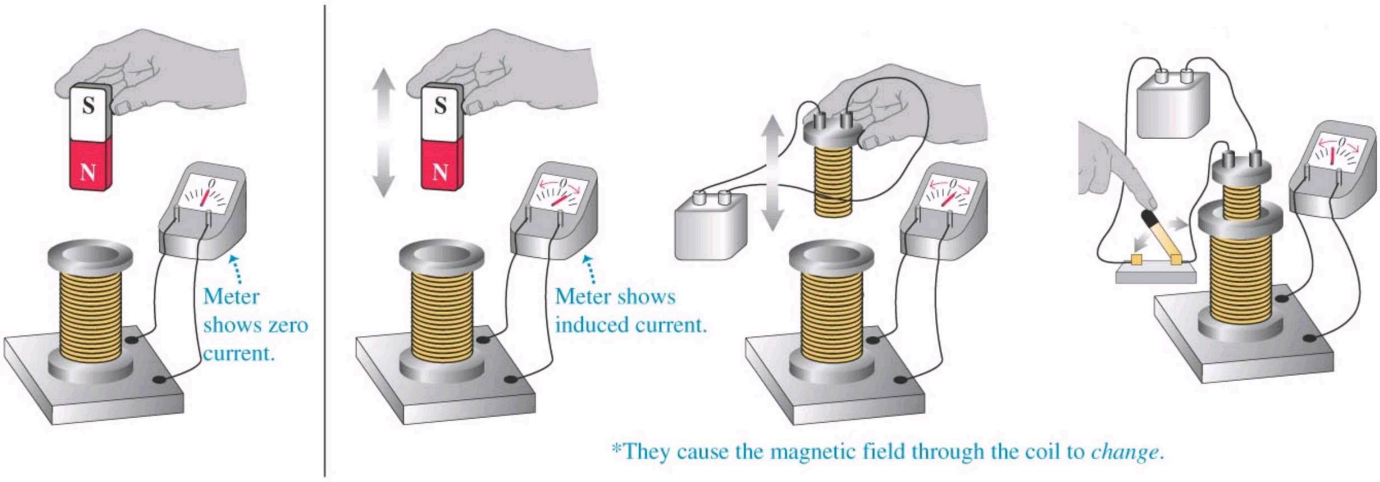
- If the magnetic flux through a circuit changes, an emf and a current are induced.

- A time-varying magnetic field can act as source of electric field.
- A time-varying electric field can act as source of magnetic field.

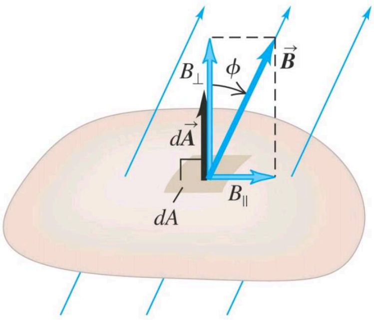
Maxwell

1. Induction Experiments (Faraday / Henry)

- An induced current (and emf) is generated when: (a) we move a magnet around a coil, (b) move a second coil toward/away another coil, (c) change the current in the second coil by opening/closing a switch.



- Magnetically induced emfs are always the result of the action of non-electrostatic forces. The electric fields caused by those forces are $\vec{E}_n$ (non-Coulomb, non conservative).

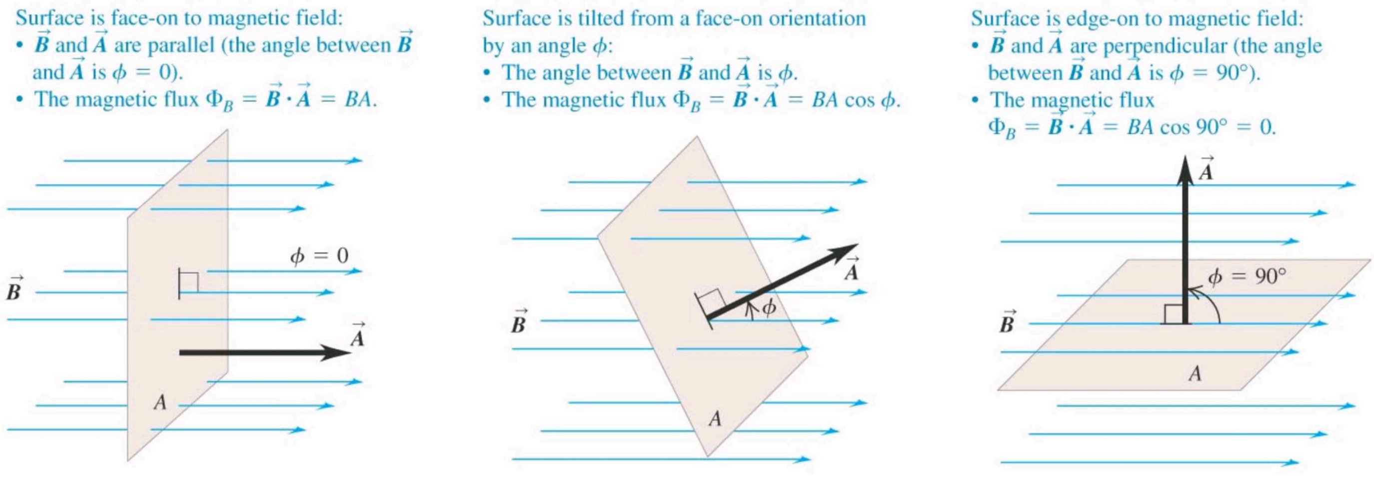


2. Faraday's Law

Magnetic flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos\varphi \cdot dA$$

If $\vec{B}$ is uniform over a flat area $\vec{A}$: $\Phi_B = \vec{B} \cdot \vec{A} = B \cdot A \cdot \cos\varphi$



Faraday's Law of Induction:

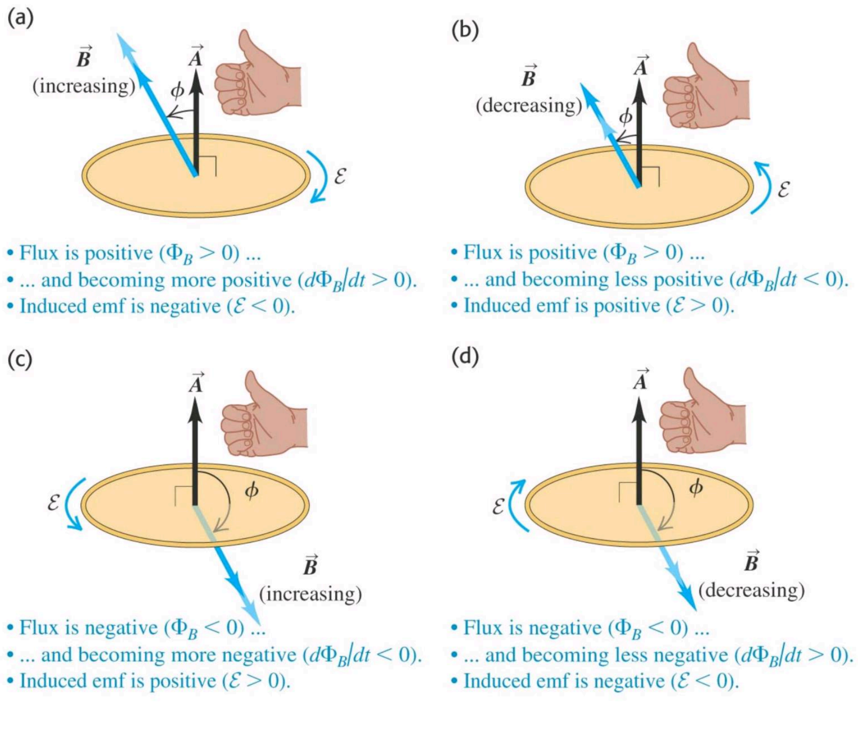
- *The induced emf in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop.*

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- Increasing flux → ε < 0 ;   Decreasing flux → ε > 0

- <u>Direction:</u> curl fingers of right hand around $\vec{A}$, if ε > 0 is in same direction of fingers (counter-clockwise), if ε < 0 contrary direction (clockwise).

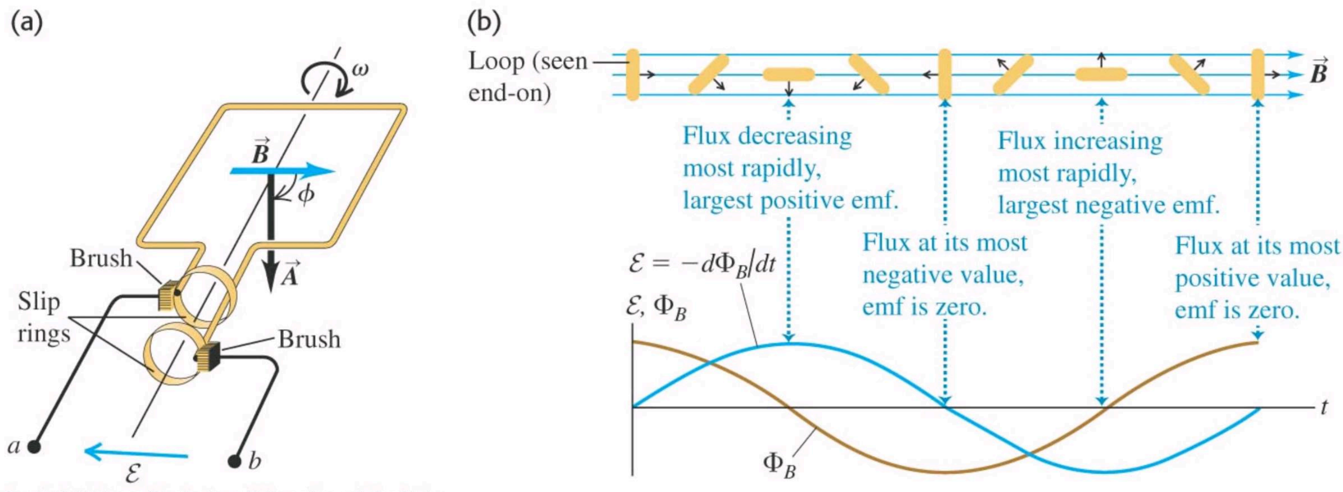
- Only a change in the flux through a circuit (not flux itself) can induce emf. If flux is constant → no induced emf.



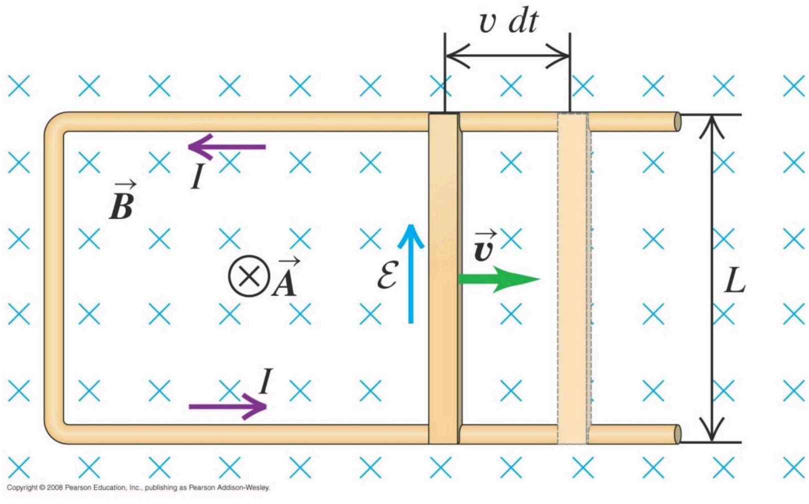
Coil: $$\varepsilon = -N\frac{d\Phi_B}{dt}$$ N = number of turns

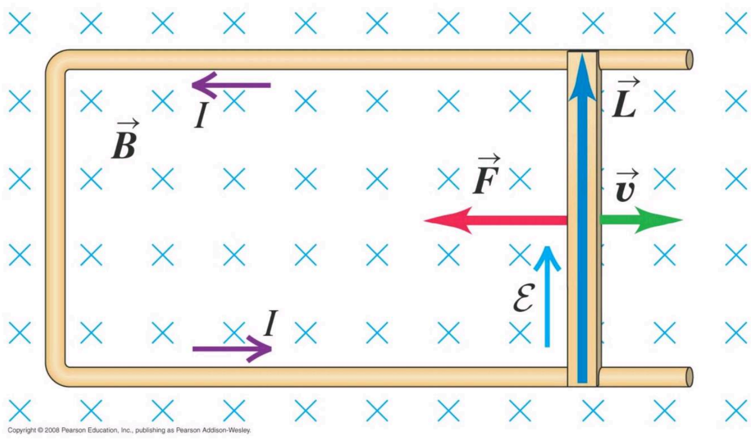
- If the loop is a conductor, an induced current results from emf. This current produces an additional magnetic field through loop. From right hand rule, that field is opposite in direction to the increasing field produced by electromagnet.

Ex: 29.4 - Generator I: a simple alternator



Exs: 29.6, 29.7 - Generator III: the slide wire generator



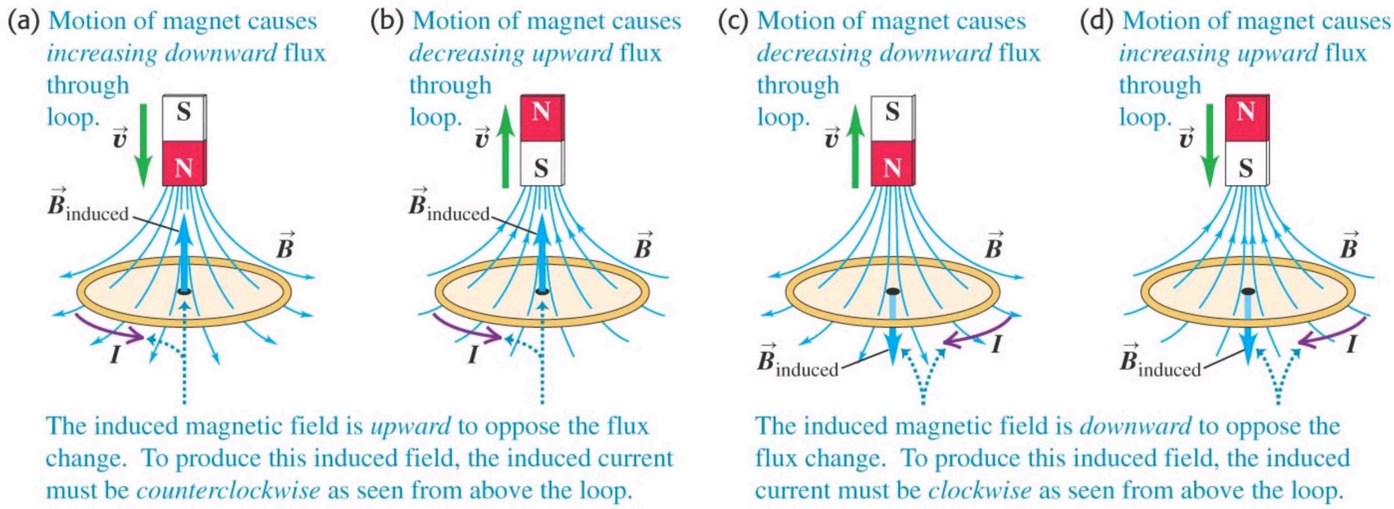


3. Lenz's Law

- Alternative method for determining the direction of induced current or emf.

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

-The "cause" can be changing the flux through a stationary circuit due to varying B, changing flux due to motion of conductors, or both.

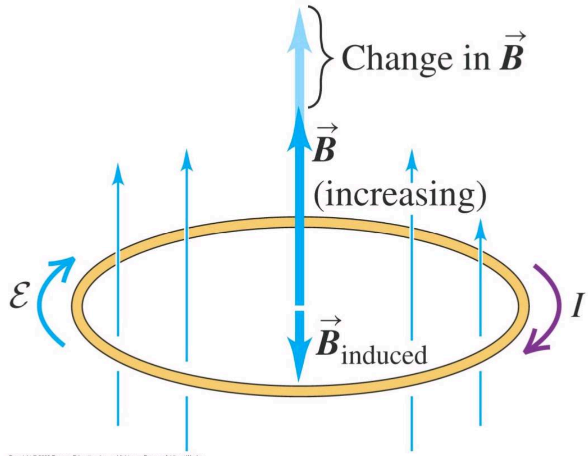


- If the flux in an stationary circuit changes, the induced current sets up a magnetic field *opposite* to the original field if original B *increases*, but in the *same direction* as original B if B *decreases*.

- The induced current opposes the change in the flux through a circuit (not the flux itself).

- If the change in flux is due to the motion of a conductor, the direction of the induced current in the moving conductor is such that the direction of the magnetic force on the conductor is opposite in direction to its motion (e.g. slide-wire generator). The induced current tries to preserve the "status quo" by opposing motion or a change of flux.

B induced downward opposing the change in flux (dΦ/dt). This leads to induced current clockwise.



Lenz's Law and the Response to Flux Changes

- Lenz's Law gives only the *direction* of an induced current. The *magnitude* depends on the circuit's resistance. Large R → small induced I → easier to change flux through circuit.

- If loop is a good conductor → I induced present as long as magnet moves with respect to loop. When relative motion stops → I = 0 quickly (due to circuit's resistance).

- If R = 0 (superconductor) → I induced (*persistent current*) flows even after induced emf has disappeared (after magnet stopped moving relative to loop). The flux through loop is the same as before the magnet started to move → flux through loop of R =0 does not change.

Magnetic levitation:

-The principle of levitation is Lenz' rule.

1) The magnetic field created by the induced current in a metallic sample due to time-fluctuation of the external magnetic field of the coil wants to avoid its cause (i.e., the coil's fluctuating magnetic field).

2) Thus, the induced magnetic field in the sample and the external fluctuating magnetic field of the coil repel each other.

3) The induced magnetic field (and the sample) move away from its cause, i.e. away from the coil's magnetic field. Then, for a conical coil (smaller radius at the bottom than at the top) the metallic sample will move upward due to this levitation force, until the force of gravity balances the force of levitation. (The levitation force is larger at the bottom of the conical coil than at the top of the coil).

Induced Current / Eddy current levitation:

- The rail and the train exert magnetic fields and the train is levitated by repulsive forces between these magnetic fields.
- B in the train is created by electromagnets or permanent magnets, while the repulsive force in the track is created by a induced magnetic field in conductors within the tracks.
- Problems:

(1) at slow speeds the current induced in the coils of the track's conductors and resultant magnetic flux is not large enough to support the weight of the train. Due to this, the train needs wheels (or any landing gear) to support itself until it reaches a speed that can sustain levitation.

(2) this repulsive system creates a field in the track (in front and behind the lift magnets) which act against the magnets and creates a "drag force". This is normally only a problem at low speed.

4. Motional Electromotive Force

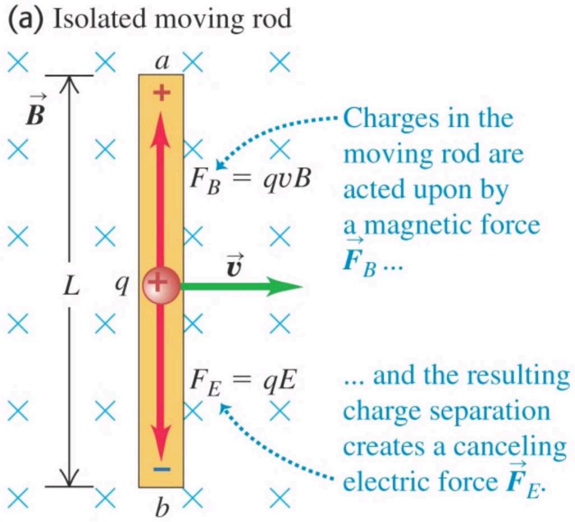
- A charged particle in rod experiences a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ that causes free charges in rod to move, creating excess charges at opposite ends.

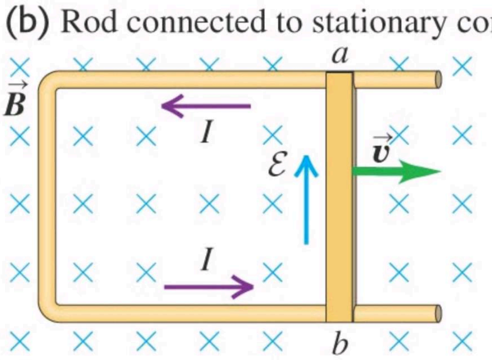
- The excess charges generate an electric field (from a to b) and electric force (F = q E) opposite to magnetic force.

- Charge continues accumulating until F_E compensates F_B and charges are in equilibrium → q E = q v B

$$V_{ab} = E \cdot L = v \cdot B \cdot L$$

- If rod slides along stationary U-shaped conductor → no F_B acts on charges in U-shaped conductor, but excess charge at ends of straight rod redistributes along U-conductor, creating an electric field.





The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

-The electric field in stationary U-shaped conductor creates a current → moving rod became a source of emf (motional electromotive force). Within straight rod charges move from lower to higher potential, and in the rest of circuit from higher to lower potential.

$$\varepsilon = vBL$$

Length of rod and velocity perpendicular to $\vec{B}$.

Induced current: $$I = \frac{\varepsilon}{R} = \frac{vBL}{R}$$

- The emf associated with the moving rod is equivalent to that of a battery with positive terminal at *a* and negative at *b*.

Motional emf: general form (alternative expression of Faraday's law)

$$d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\boxed{\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}}$$

Closed conducting loop

-This expression can only be used for problems involving <u>moving conductors</u>. When we have stationary conductors in changing magnetic fields, we need to use: $\varepsilon = -d\Phi_B/dt$.

5. Induced Electric Fields

- An induced emf occurs when there is a changing magnetic flux through a stationary conductor.

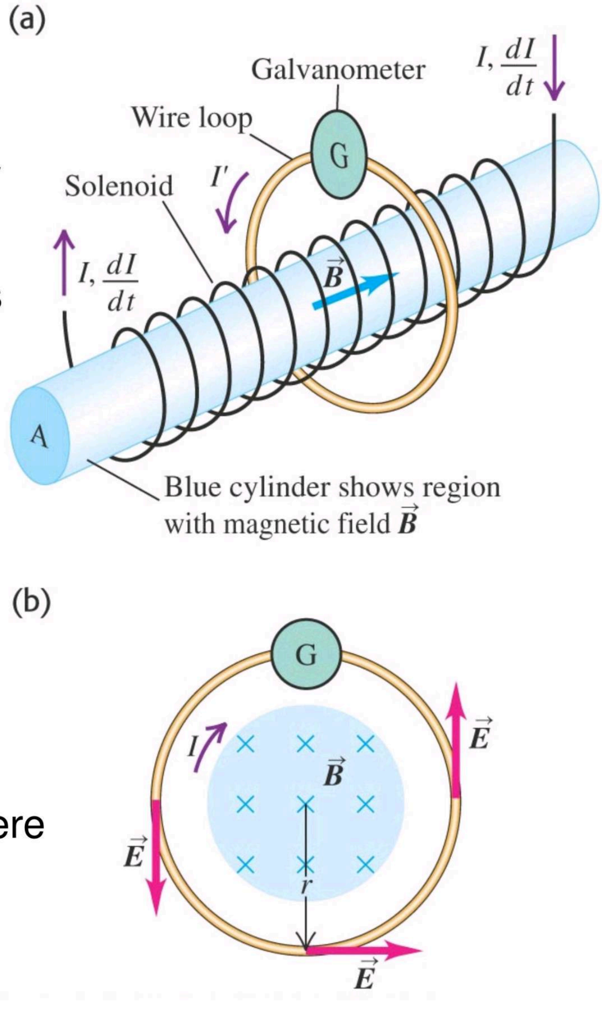
- A current (I) in solenoid sets up B along its axis, the magnetic flux is:

$$\Phi_B = B \cdot A = \mu_0 n I A$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$

Induced current in loop (I'): I' = ε / R

- The force that makes the charges move around the loop is not a magnetic force. There is an induced electric field in the conductor caused by a changing magnetic flux.



-The total work done on q by the induced $\vec{E}$ when it goes once around the loop: W = q ε → $\vec{E}$ is not conservative.

Conservative $\vec{E}$ → $\oint \vec{E} \cdot d\vec{l} = 0$

Non-conservative $\vec{E}$ → $\oint \vec{E} \cdot d\vec{l} = \varepsilon = -\frac{d\Phi_B}{dt}$ (stationary integration path)

- Cylindrical symmetry → $\vec{E}$ magnitude constant, direction is tangent to loop.

$$\oint \vec{E} \cdot d\vec{l} = 2\pi \cdot r \cdot E \longrightarrow E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

- Faraday's law: 1) an emf is induced by magnetic forces on charges when a conductor moves through $\vec{B}$.

2) a time-varying $\vec{B}$ induces $\vec{E}$ in stationary conductor and emf. $\vec{E}$ is induced even when there is no conductor. Induced $\vec{E}$ is non-conservative, "non-electrostatic". No potential energy associated, but $\vec{F}_E = q\,\vec{E}$.

6. Eddy Currents

- Induced currents that circulate throughout the volume of a material.

Ex.: $\vec{B}$ confined to a small region of rotating disk → Ob moves across $\vec{B}$ and emf is induced → induced circulation of eddy currents.
Sectors Oa and Oc are not in B, but provide return conducting paths for charges displaced along Ob to return from b to O.

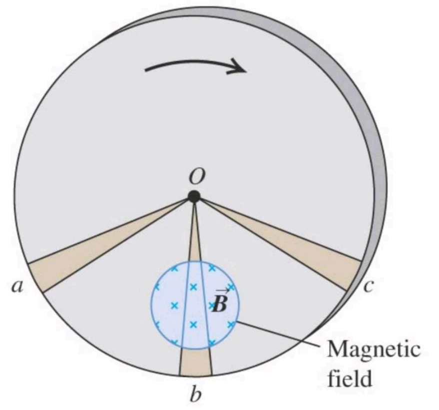
Induced I experiences $\vec{F}_B$ that opposes disk rotation:

$\vec{F} = I\vec{L} \times \vec{B}$ (right) → current and $\vec{L}$ downward.

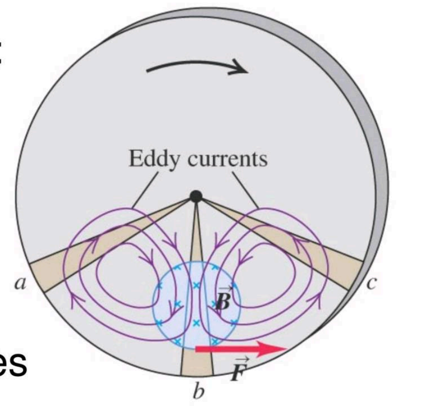
(the return currents lie outside $\vec{B}$ → do not experience F_B).

-The interaction between eddy currents and B causes braking of disk.

(a) Metal disk rotating through a magnetic field



(b) Resulting eddy currents and braking force



CHAPTER 5

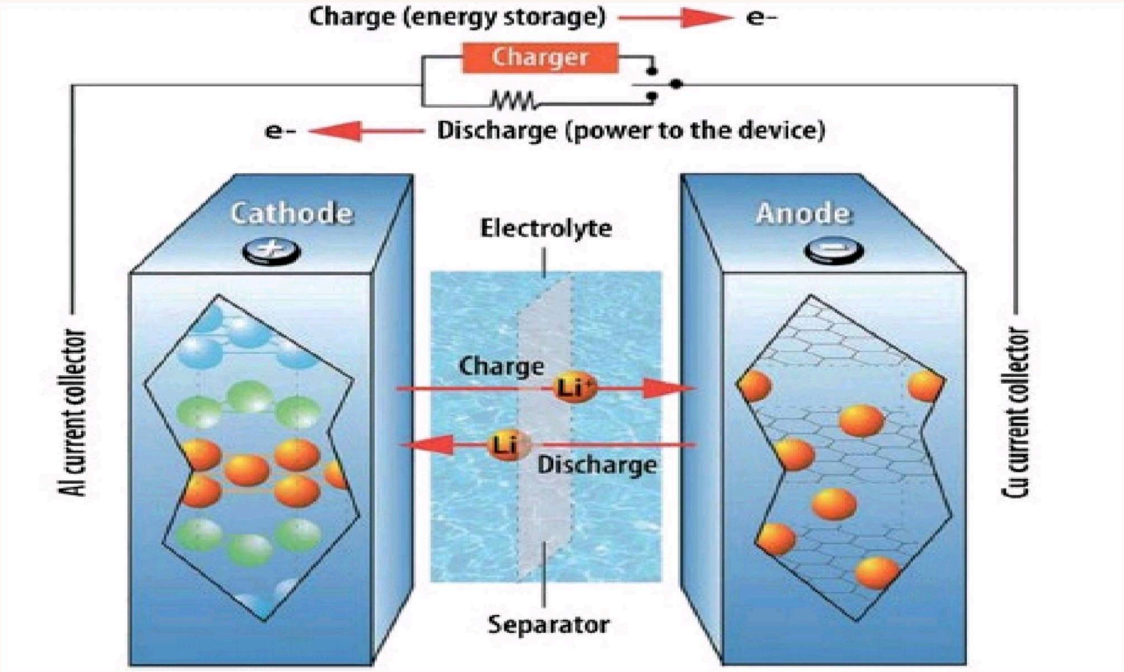
BATTERY

Batteries

- Definition
  - A battery is a storage device used for the storage of chemical energy and for the transformation of chemical energy into electrical energy
  - Batteries consist of groups of two or more electric cells connected either serial for voltage or in parallel for current.
- Two major types:
  - Primary batteries
  - Secondary batteries

Components of a typical battery

1. Cathode
2. Anode
3. Electrolyte

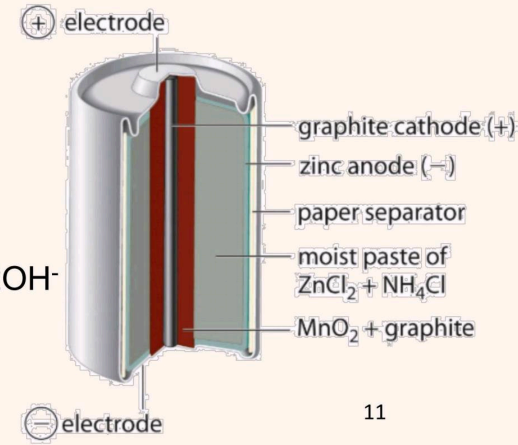


From: M. Osiak, et al," J. Mater. Chem. A, 2014, 2, 9433–9460

Primary Batteries

- Definition
  - In primary cells, the chemical reaction occurs only once
  - Source of direct current power
- Voltage ranging from 1.25-1.50 V, used in torches, radios, watches etc.
- Low-cost, but short life

- Zn-MnO_2 cell (dry cell)
  - Oxidation at anode: $Zn(s) \rightarrow Zn^{+2}(aq)+2e^-$
  - Reduction at cathode: $2MnO_2(s)+H_2o+2e^- \rightarrow Mn_2O_3(s)+2OH^-$
  - Voltage: 1.5 V

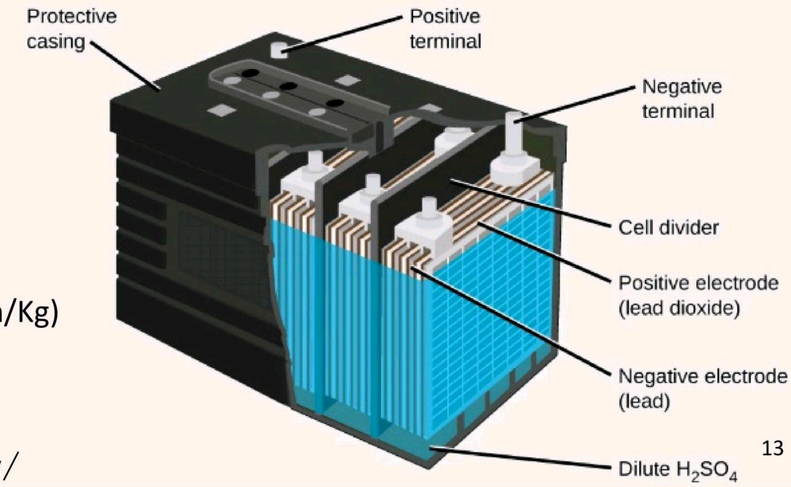


Secondary Batteries

- Definition
  - In secondary cells, the chemical reactions are reversible
  - These cells can be recharged by electric current
- Secondary cells are widely used in electric and hybrid vehicles, smart phones, digital cameras, laptops, etc.

Lead acid battery

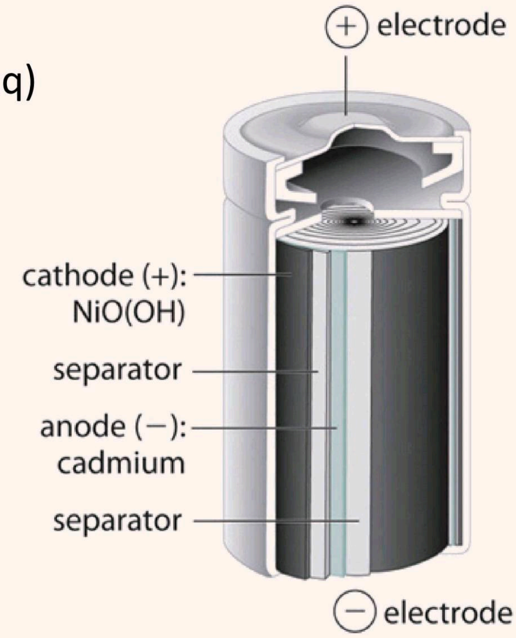
- At anode: Pb (s) + SO_4^{2-} (aq.) → $PbSO_4$ (s) + $2e^-$
- At cathode: PbO_2 (s) + SO_4^{2-} (aq.) + $4H^+$ (aq.) + $2e^-$ → $PbSO_4$ (s) + $2H_2O$
- Voltage: 2 V, used for railways, power stations (stand-by supplies)
- Advatages:
  - Reliable - constant potential
  - Inexpensive
  - Rechargeable, portable
- Drawbacks:
  - Low life cycle
  - Low energy density (30 ~ 40 Wh/Kg)
  - Use of H_2SO_4 is dangerous



https://byjus.com/chemistry/lead-acid-battery/

Nickel–cadmium cell

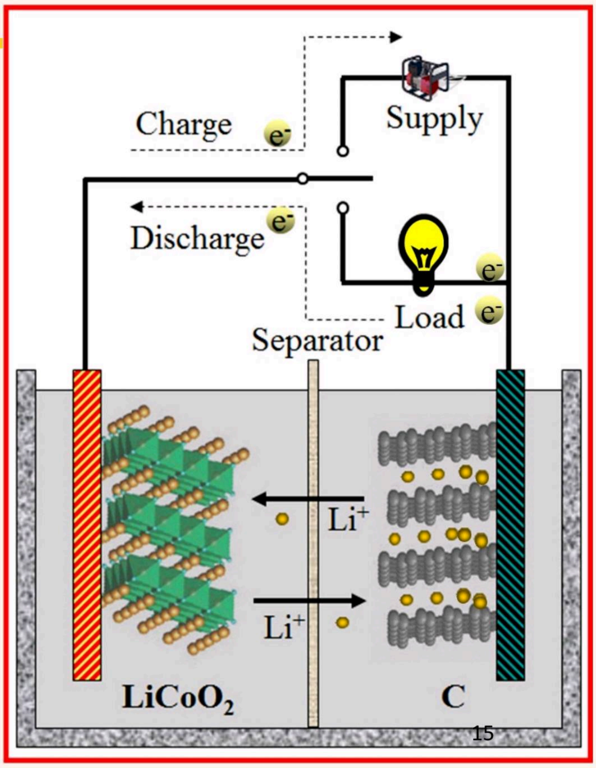
- At Anode: Cd (s) + 2OH$^-$ (aq) → Cd(OH)2 (s) + 2e$^-$
- At Cathode: NiO(OH) (s) + $2H_2O$ + 2e$^-$ →2 $Ni(OH)_2$ + OH$^-$(aq)
- Voltage: 1.4V
- In small electronic calculators, electronic flash units, etc.
- Advantages:
  - Rechargeable
  - High energy density
- Drawbacks:
  - Self discharge (20-30 % per month)
  - Memory effect
  - Toxicity of cadmium



https://chem.libretexts.org/Bookshelves/General_Chemistry/Map%3A_Chemistry_(Zumdahl_and_Decoste)/11%3A_Electrochemistry/11.5%3A_Batteries

Li Ion Battery

- At anode: $xLi^+ + xe^- + 6C \leftrightarrow Li_xC_6$
- At cathode: $LiCoO_2 \leftrightarrow Li_{1-x}CoO_2 + xLi^+ + xe^-$
- Compared to Ni-Cd batteries:
  - No memory effect
  - Higher voltages (~3 ×)
  - Self discharge <5 % per month



D. Deng, *Energy Science & Engineering*, **2015**, 3, 5, 385-418