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Structural Mechanics

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STRESS :

Stresses are expressed as the ratio of the applied force divided by the resisting area

Mathematically:

σ = Force / Area

Units:

N/m^2 or Pascal.

1kPa = 1000Pa, 1 Mpa= 10^6 Pa

TERMINOLOGIES RELATED TO STRESS

- **Stressor:**

A stressor is anything that has the effect of causing stress.

- **Stress capacity:**

While it is unclear precisely how much stress a person can carry, since each person has some stress in their lives, we say he/she has a capacity for stress. Similarly in case of Rocks, how much capacity they have to bear stress.

- **Stress-load:**

Everyone, even children, must carry some amount of stress in their daily lives. When we think of stress as having an amount, or quantity, we refer to this as the person's stress-load. And here in case of rocks, we say that how much an already existing stress is applied on a rock.

TYPES OF STRESS

There are two types of stress

1) **Normal Stress**

1) **Tensile stress**

2.2) **Compressive stress**

2) **Combine Stress**

1) **Shear stress**

2) **Tortional stress**

1)Normal Stress:

The resisting area is perpendicular to the applied force

1.1) Tensile Stress:

- **It is a stress induced in a body when it is subjected to two equal and opposite pulls (Tensile force) as a result of which there is tendency in increase in length.**
- **It acts normal to the area and pulls on the area.**

1.2) Compressive Stress:

- **Stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a tendency of decrease in length of the body.**
- **It acts normal to the area and it pushes on the area.**

2) Combined Stress:

- A condition of stress that cannot be represented by a single resultant stress.

- 2.1) Shear stress:

❖ Forces parallel to the area resisting the force cause shearing stress.

❖ It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act.

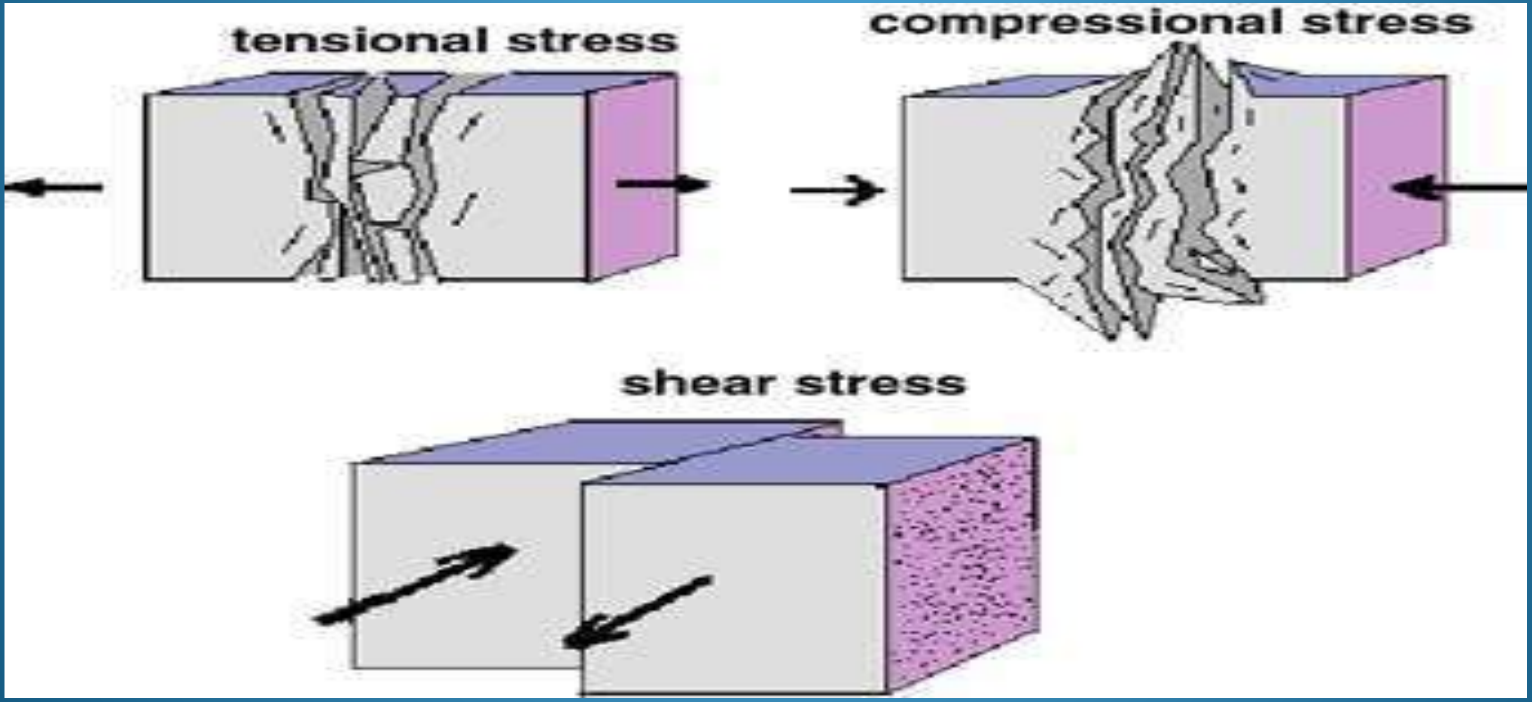
❖ Shearing stress is also known as tangential stress

- .

- 2.2) Tortional stress:

TYPES OF STRESS:

Diagrams



STRAIN:

When a body is subjected to some external force, there is some change in the dimension of the body. The ratio of change in dimension of body to its original dimension is called as strain.

Strain is a dimensionless quantity.

TERMINOLOGIES RELATED TO STRAIN:

- **Longitudinal or Linear Strain**

  Strain that changes the length of a line without changing its direction. Can be either compressional or tensional.

- **Compression**

  Longitudinal strain that shortens an object.

- **Tension**

  Longitudinal strain that lengthens an object.

➢Shear

❖ Strain that changes the angles of an object.

❖Shear causes lines to rotate.

➢Infinitesimal Strain

❖ Strain that is tiny, a few percent or less.

❖ Allows a number of useful mathematical simplifications and approximations.

➢Finite Strain

❖ Strain larger than a few percent.

❖Requires a more complicated mathematical treatment than infinitesimal strain.

➢Homogeneous Strain

- Uniform strain.
- Straight lines in the original object remain straight.
- Parallel lines remain parallel.
- Circles deform to ellipses.
- Note that this definition rules out folding, since an originally straight layer has to remain straight.

➢Inhomogeneous Strain

- How real geology behaves.
- Deformation varies from place to place.
- Lines may bend and do not necessarily remain parallel.

TYPES OF STRAIN

1. Tensile Strain
2. Compression Strain
3. Volumetric Strain
4. Shear Strain

1) Tensile Strain:

Ratio of increase in length to the original length of the body when it is subjected to a pull force.

Tensile strain = Increase in length/ Original Length

= dL/L

2) Compressive Strain:

Ratio of decrease in Length to the original length of body when it is subjected to push force.

Compressional Strain = Decrease in length/Original Length

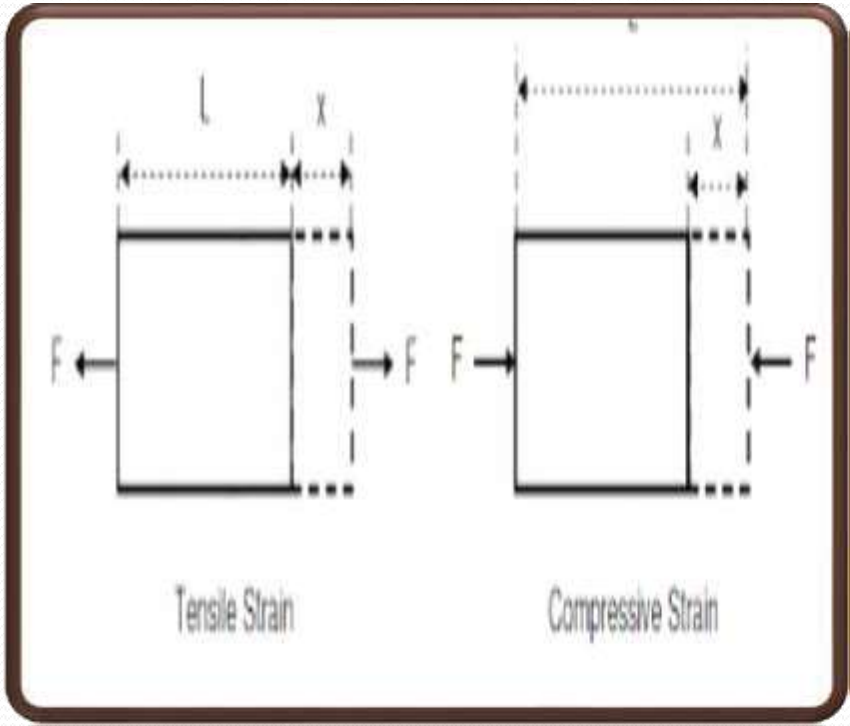
= dL/L

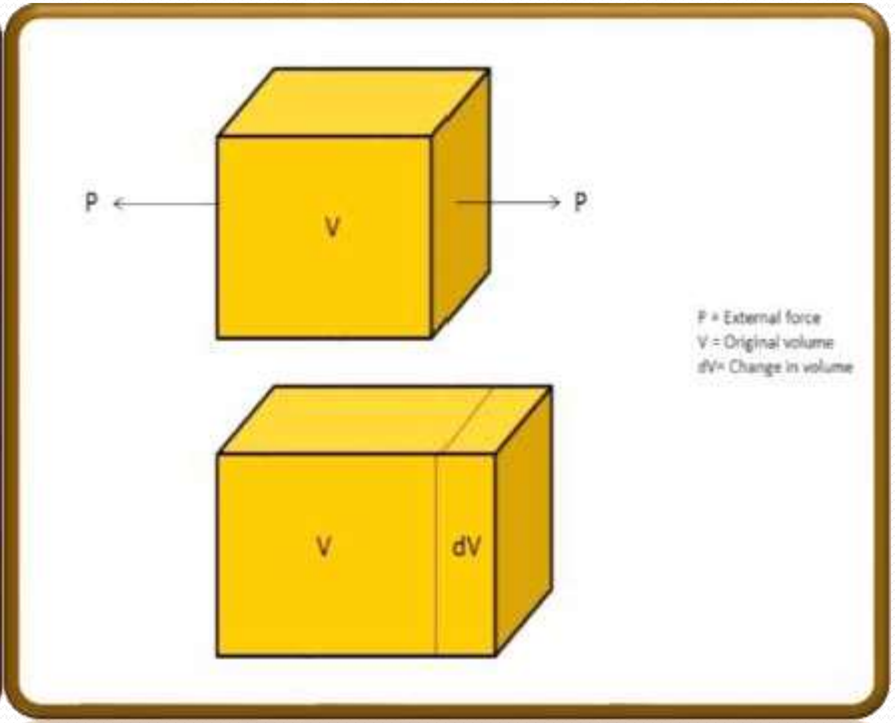
3) Volumetric Strain:

Ratio of change of volume to the original volume.

Volumetric Strain= dV/V

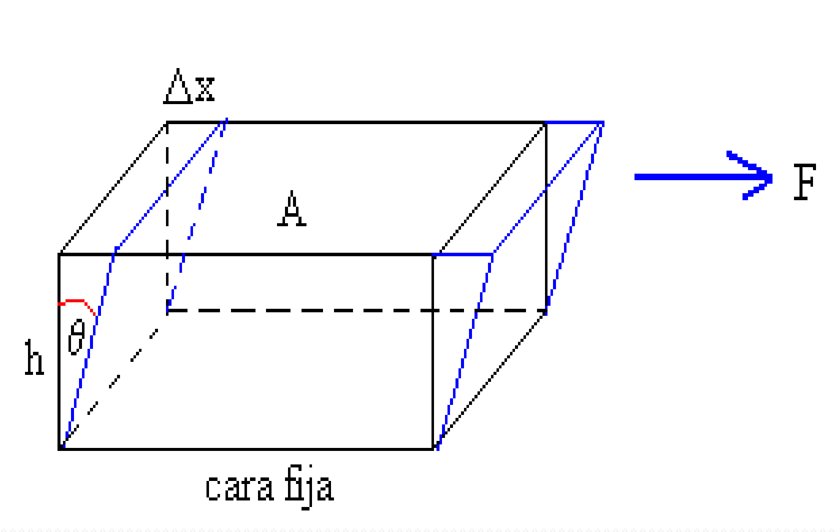
Types of Strain
Diagrams





4) Shear Strain

Strain due to shear stresses.



Sign convection for direct strain

- ❖Tensile strains are considered positive in case of producing increase
- ❖ in length.
- ❖Compressive strains are considered negative in case of producing
- ❖decrease in length.

Relation between Stress and Strain

Hooke's Law:

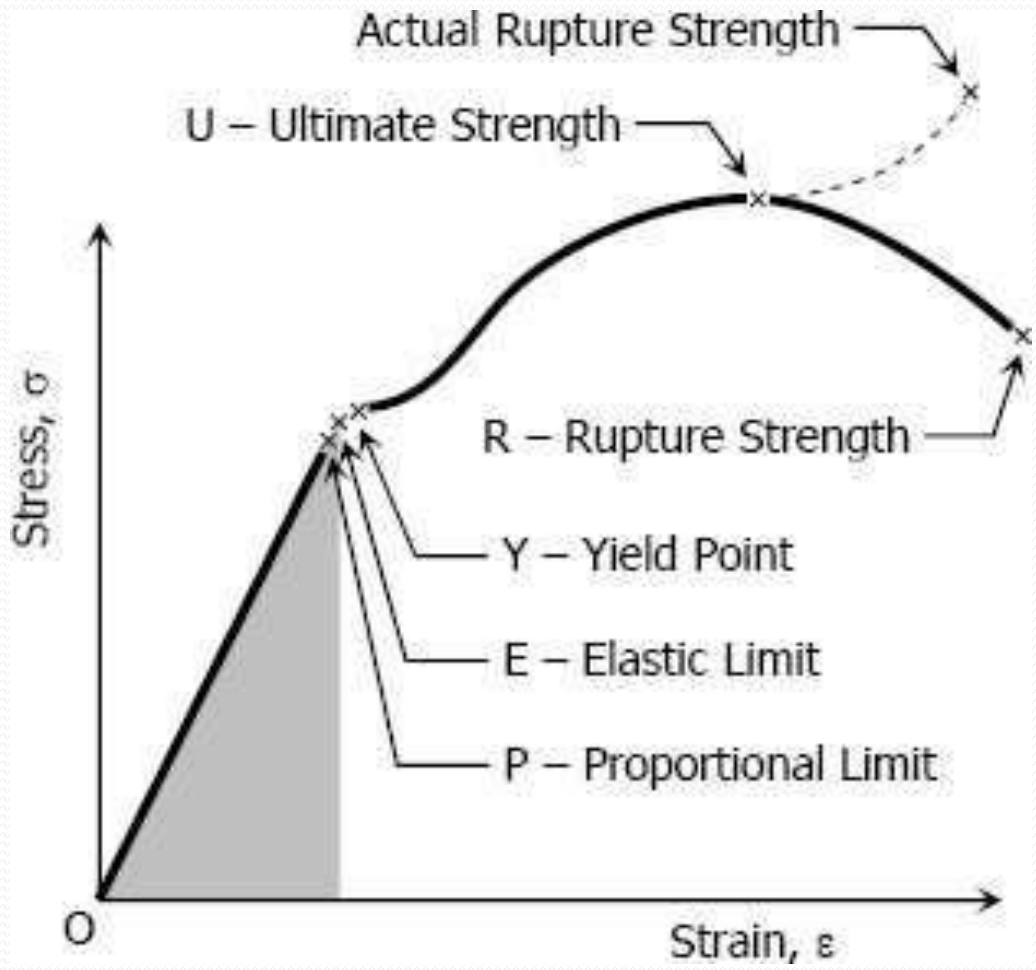
"Within Elastic limit the ratio of stress applied to strain developed is constant".
The constant is known as Modulus of elasticity or Young's Modulus or Elastic Modulus.

Mathematically:

E= Stress/Strain

- ❖**Young's Modulus E, is generally assumed to be same in tension or Compression and for most of engineering application has high Numerical value.**
- ❖**Typically E=210×10*9 N/m*2 for steel**

STRESS AND STRAIN DIAGRAM

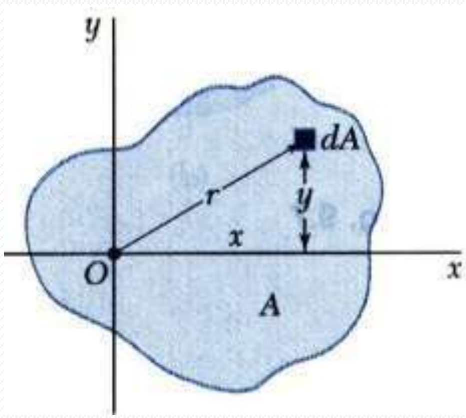


Moment of Inertia

1. Definition
2. Standard shape derivation
3. Moments of Inertia of Composite Areas
4. Parallel Axis Theorem
5. Some examples of M.I

1. Definition

- Moment of inertia is second moment of area.



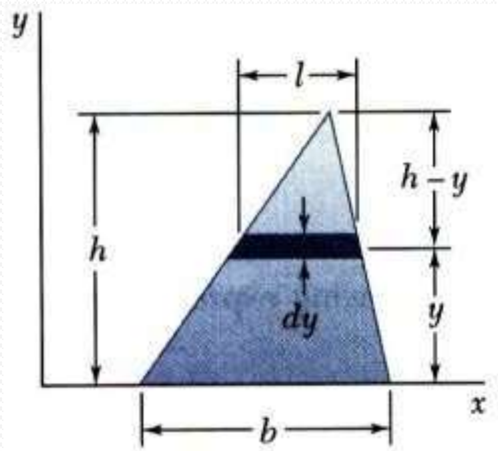
- Consider a lamina of area A
- Let this lamina is split up in to an infinite of small elements even of area da..
- x1,x2,x3……are distance of small element from OY axis.
- y1,y2,y3……are distance of small elements from OX axis.

- Taking secound moment of all the small elements about OX axis

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

- Thus secound moment of area is called moment of inertia.
- UNIT : mm4, cm4

•Standerd shape darivation



Determine the moment of inertia of a triangle with respect to its base.

Same as rectangle..

SOLUTION:

- A differential strip parallel to the *x* axis is chosen for *dA.*

$$I_x = \int y^2 \, dA$$

$$dA = l \, dy$$

- For similar triangles,

$$\frac{l}{b} = \frac{h-y}{h} \qquad l = b\frac{h-y}{h} \qquad dA = b\frac{h-y}{h}dy$$

Integrating dI_x from $y = 0$ to $y = h$

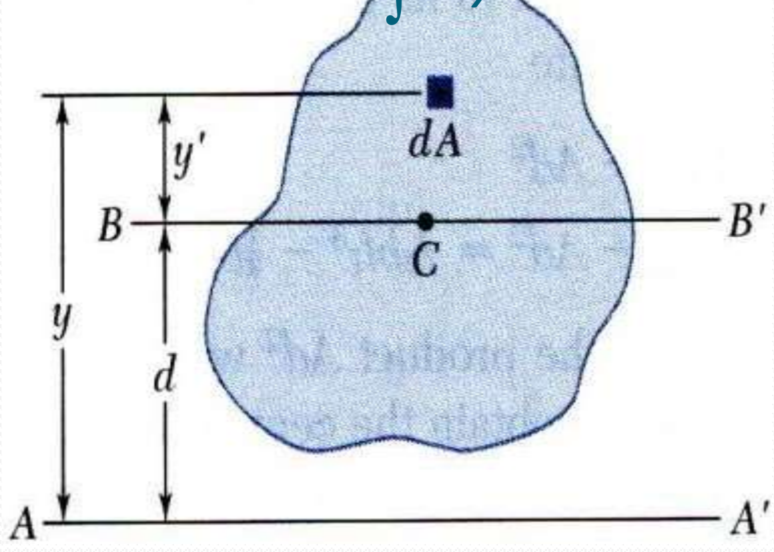
$$I_x = \int y^2 dA = \int_0^h y^2 b \frac{h-y}{h} dy = \frac{b}{h}\int_0^h \left(hy^2 - y^3\right) dy$$

$$= \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h$$

$$\boxed{I_x = \frac{bh^3}{12}}$$

•Parallel Axis Theorem

$$I = \int y^2 \, dA$$



Consider moment of inertia I with respect to the axis AA'.

This axis BB' passes through the area centroid & is called a centroidal axis..

$$I = \int y^2 dA = \int (y' + d)^2 dA$$

$$= \int y'^2 dA + 2d \int y' dA + d^2 \int dA$$

$$I = \bar{I} + A d^2$$

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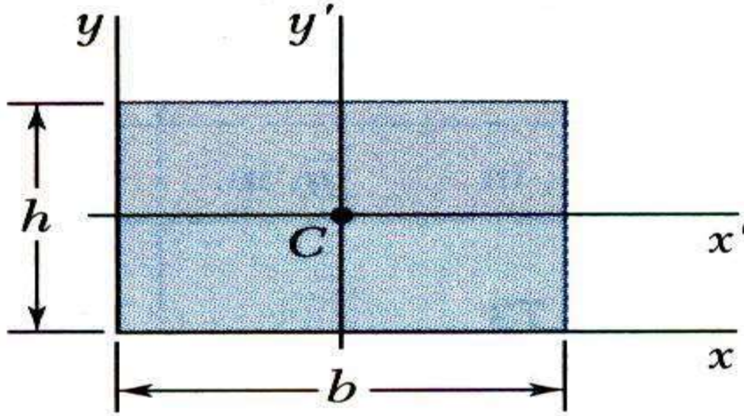
<tr>
<td>Rectangle</td>
<td>
</td>
<td>$\overline{I}_{x'} = \frac{1}{12} bh^3$
$\overline{I}_{y'} = \frac{1}{12} b^3h$
$I_x = \frac{1}{3} bh^3$
$I_y = \frac{1}{3} b^3h$
$J_C = \frac{1}{12} bh(b^2 + h^2)$</td>
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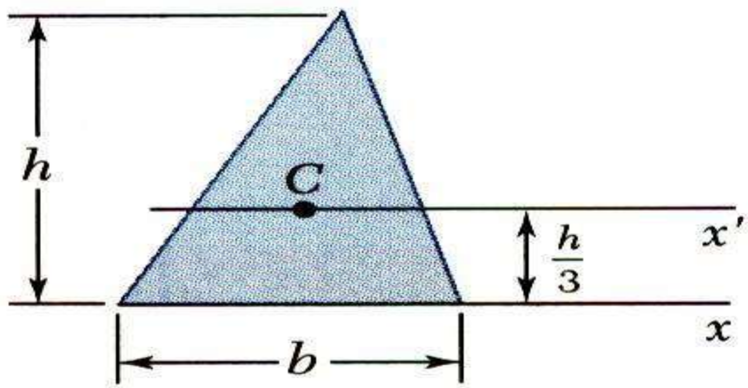
<tr>
<td>Triangle</td>
<td>
</td>
<td>$\overline{I}_{x'} = \frac{1}{36} bh^3$
$I_x = \frac{1}{12} bh^3$</td>
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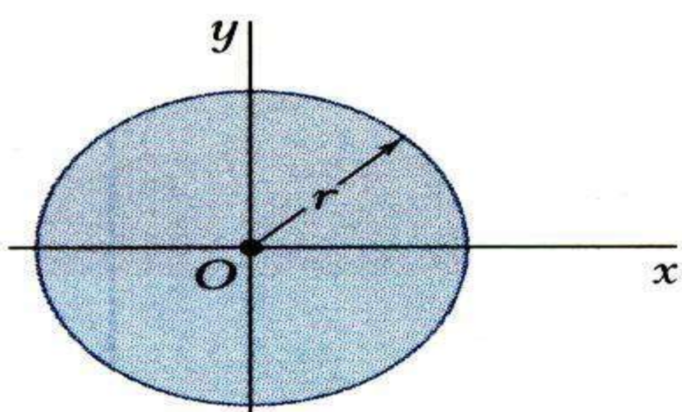
<tr>
<td>Circle</td>
<td>
</td>
<td>$\overline{I}_x = \overline{I}_y = \frac{1}{4}\pi r^4$
$J_O = \frac{1}{2} \pi r^4$</td>
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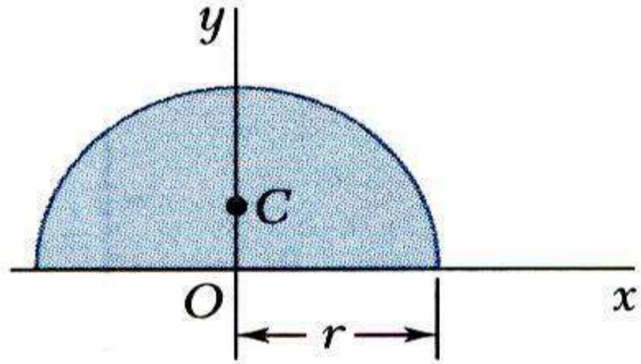
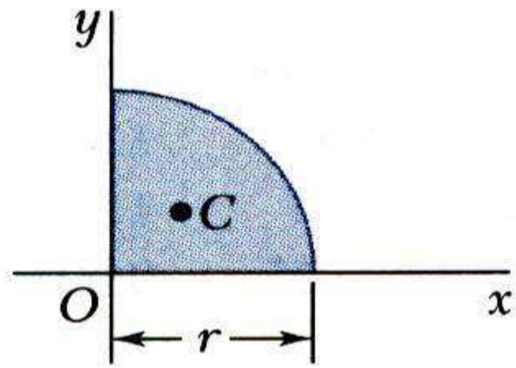
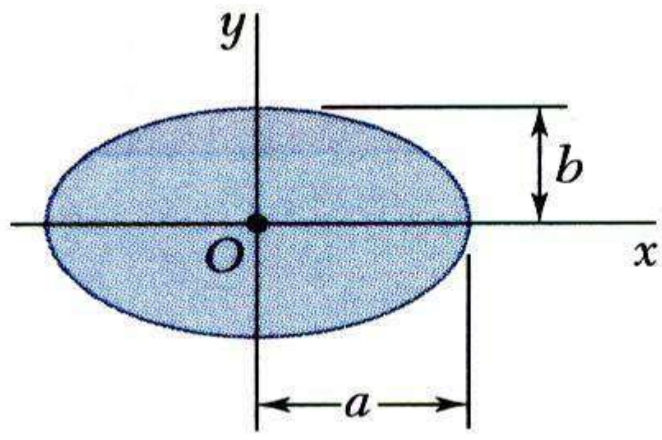
</table>

Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3 b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

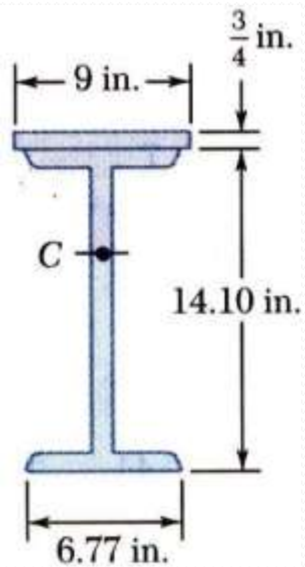
SOLUTION:

- Determine location of the centroid of composite section with respect to a coordinate system with origin at the centroid of the beam section.

- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.

The strength of a W14x38 rolled steel beam is increased by attaching a plate to its upper flange.

Determine the moment of inertia..



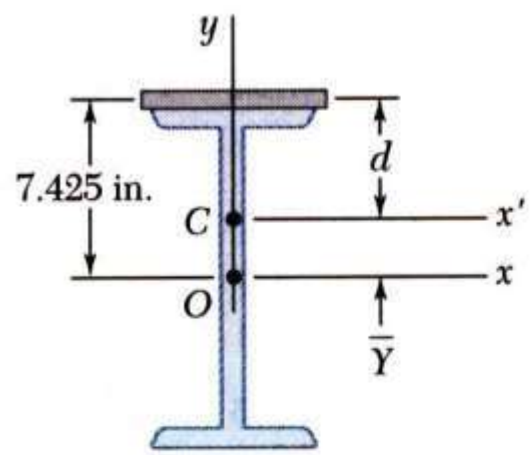
- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.

$$I_{x',\text{beam section}} = \bar{I}_x + A\bar{Y}^2 = 385 + (11.20)(2.792)^2$$

$$= 472.3\ \text{in}^4$$

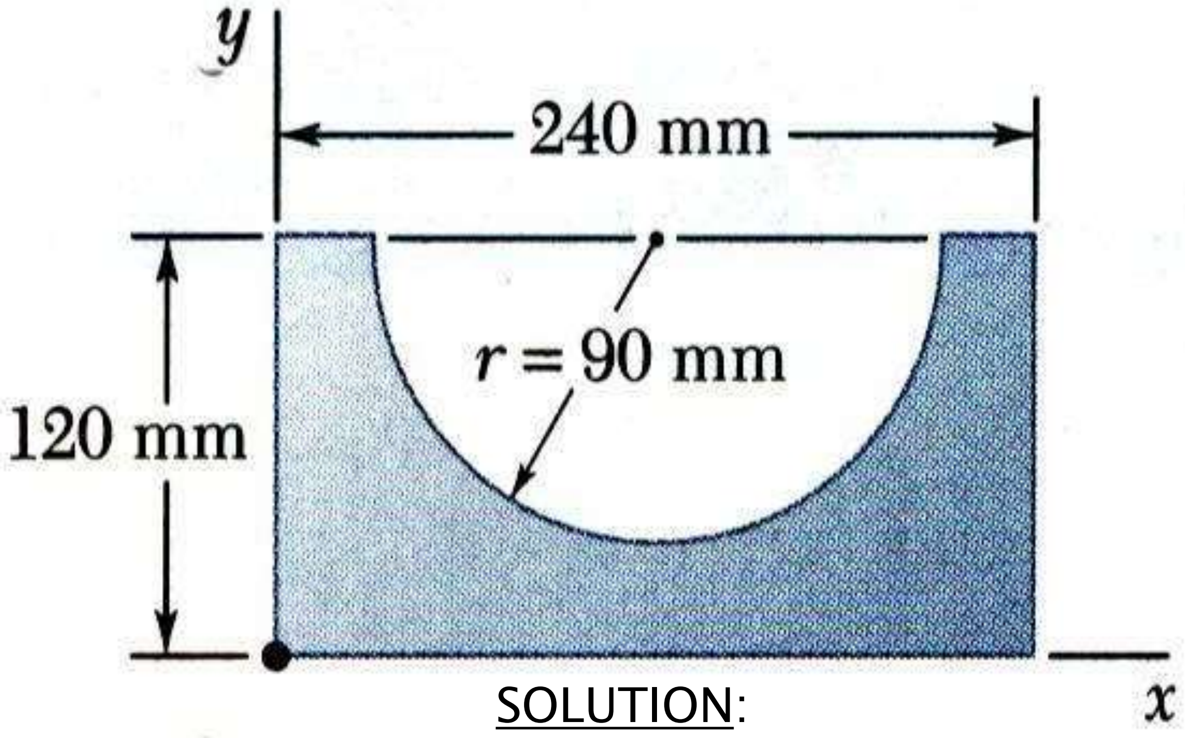
$$I_{x',\text{plate}} = \bar{I}_x + Ad^2 = \tfrac{1}{12}(9)\left(\tfrac{3}{4}\right)^3 + (6.75)(7.425 - 2.792)^2$$

$$= 145.2\ \text{in}^4$$



$$I_{x'} = I_{x',\text{beam section}} + I_{x',\text{plate}} = 472.3 + 145.2$$

$$\boxed{I_{x'} = 618\ \text{in}^4}$$

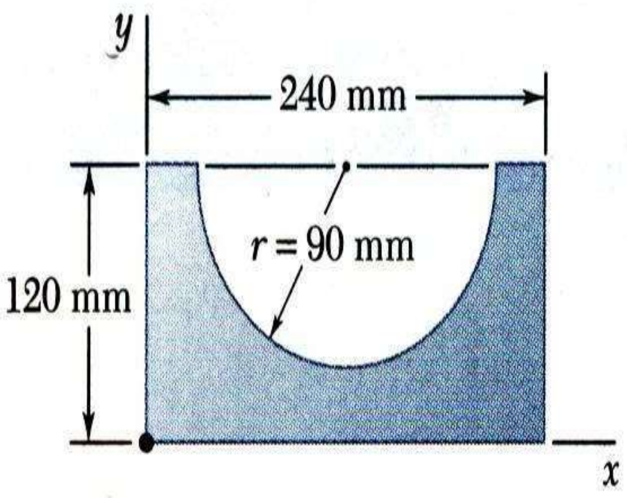


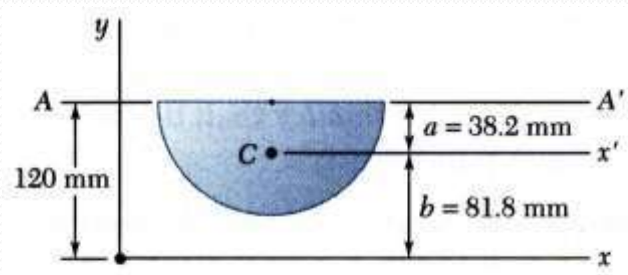
Determine the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the *x* axis.

- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.





SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the *x* axis.

Rectangle:

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240)(120) = 138.2\times10^6\,\mathrm{mm}^4$$

Half-circle:

moment of inertia with respect to *AA'*,

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90)^4 = 25.76\times10^6\,\mathrm{mm}^4$$

$$a = \frac{4r}{3\pi} = \frac{(4)(90)}{3\pi} = 38.2 \text{ mm}$$

$$\text{b} = 120 - \text{a} = 81.8 \text{ mm}$$

$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi (90)^2$$

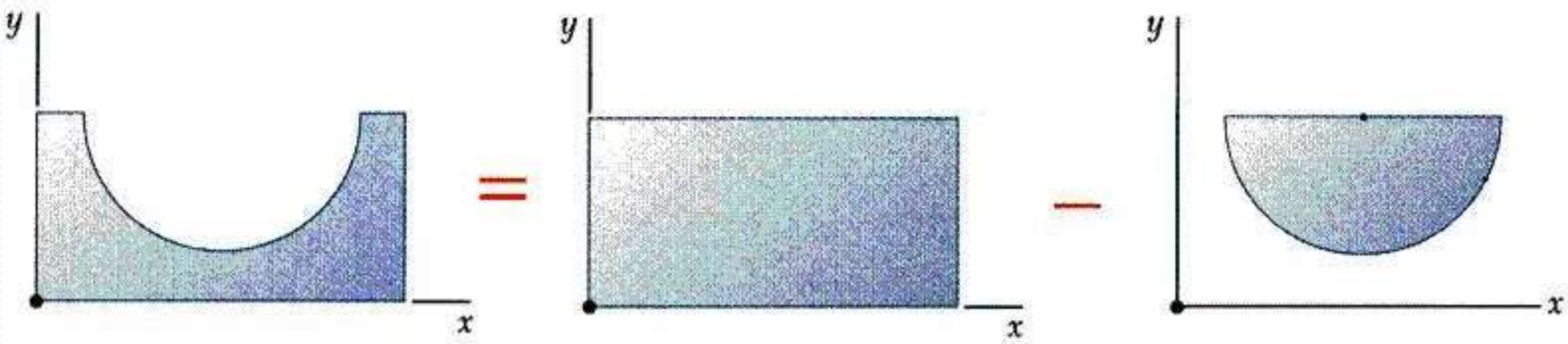
$$= 12.72 \times 10^3 \text{ mm}^2$$

moment of inertia with respect to *x′*,

$$I_{x'} = I_{AA'} - Aa^2 = (25.76 \times 10^6)(12.72 \times 10^3)$$

$$= 7.20 \times 10^6 \text{ mm}^4$$

- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.



I_x $138.2\times10^6\,\mathrm{mm}^4$ $92.3\times10^6\,\mathrm{mm}^4$

$$\boxed{I_x = 45.9\times10^6\,\mathrm{mm}^4}$$