D.C. CIRCUIT THEOREM

INTRODUCTION

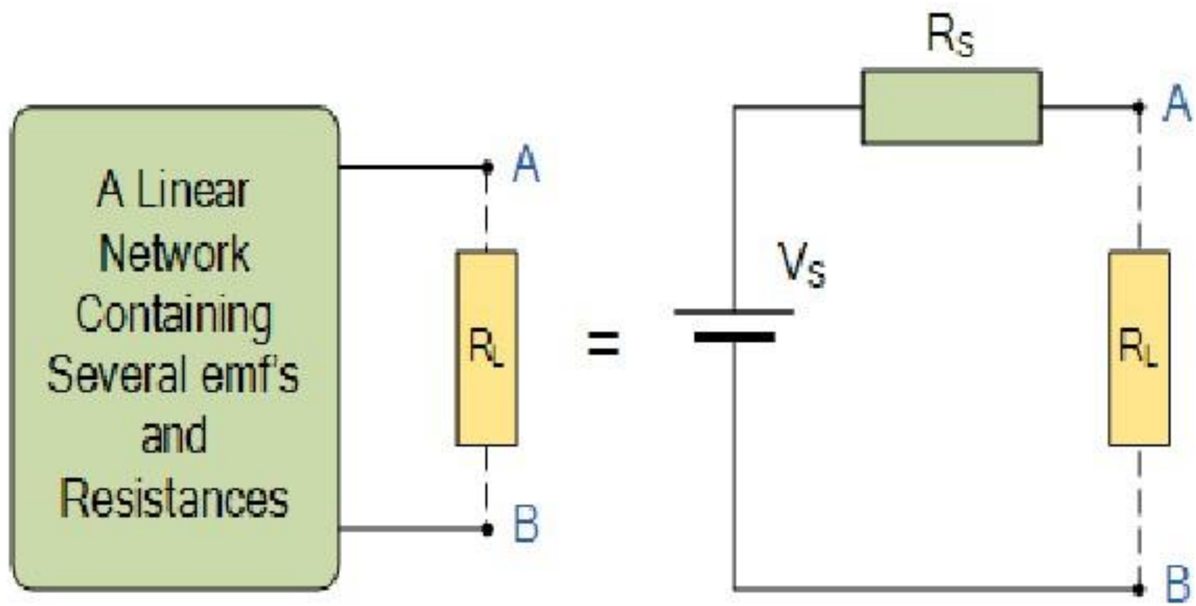
The Theorems we will look at are:

- Thevenin's and Norton's theorem, which show how to produce simple equivalent circuits, for more complicated circuits. These equivalent circuits can then be used, to more easily predict some basic characteristics of the original complex circuits.
- The superposition theorem, which shows how to find currents and voltages, in circuits that contain two or more branches and more than one e.m.f .
- For maximum power transfer having less loss of power by using maximum power transfer theorem

THEVENIN'S THEOREM

Thevenin's Theorem states that "*Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load*". In other words, it is possible to simplify any electrical circuit, no matter how complex, to an equivalent two-terminal circuit with just a single constant voltage source in series with a resistance (or impedance) connected to a load as shown below.

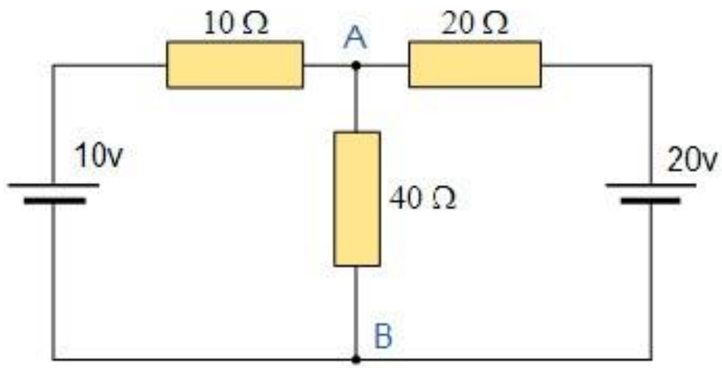
Thevenin's equivalent circuit.



As far as the load resistor R_L is concerned, any complex "one-port" network consisting of multiple resistive circuit elements and energy sources can be replaced by one single equivalent resistance Rs and one single equivalent voltage Vs. Rs is the source resistance value looking back into the circuit and Vs is the open circuit voltage at the terminals.

EXAMPLE OF THEVENIN'S THEOREM

- For example, consider the circuit from the previous section.
- Firstly, to analyze the circuit we have to remove the centre 40Ω load resistor connected across the terminals A-B, and remove any internal resistance associated with the voltage source(s). This is done by shorting out all the voltage sources connected to the circuit, that is v = 0, or open circuit any connected current sources making i = 0. The reason for this is that we want to have an ideal voltage source or an ideal current source for the circuit analysis.
- The value of the equivalent resistance, Rs is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.



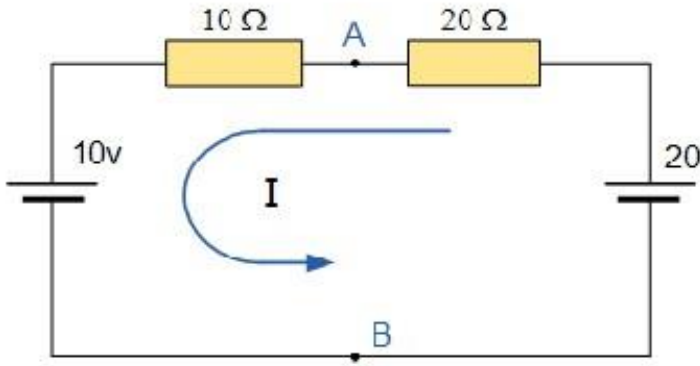
Find the Equivalent Resistance (Rs)

10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

The voltage Vs is defined as the total voltage across the terminals A and B when there is an open circuit between them. That is without the load resistor R_L connected.

Find the Equivalent Voltage (Vs)



We now need to reconnect the two voltages back into the circuit, and as $V_S = V_{AB}$ the current flowing around the loop is calculated as:

$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$

This current of 0.33 amperes (330mA) is common to both resistors so the voltage drop across the 20Ω resistor or the 10Ω resistor can be calculated as:

V_{AB} = 20 - (20Ω x 0.33amps) = 13.33 volts. or

V_{AB} = 10 + (10Ω x 0.33amps) = 13.33 volts, the same.

Then the Thevenin's Equivalent circuit would consist or a series resistance of 6.67Ω's and a voltage source of 13.33v. With the 40Ω resistor connected back into the circuit we get:

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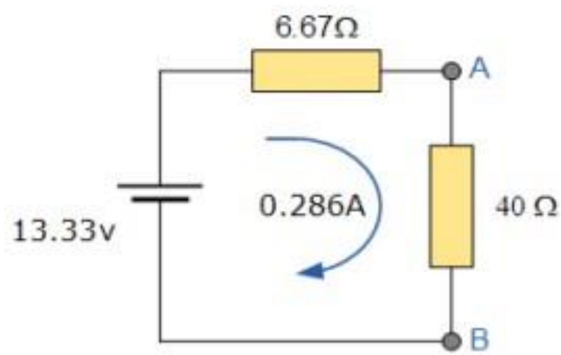


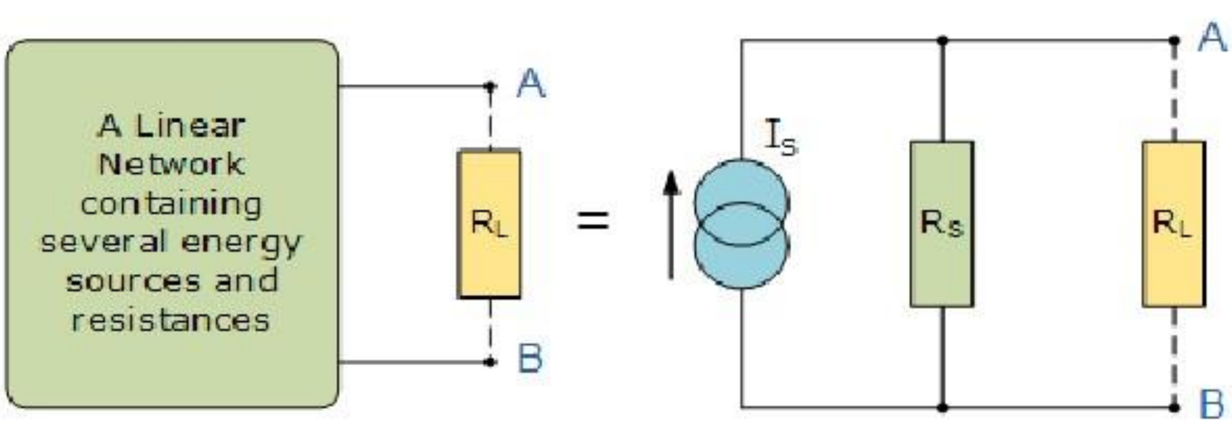
and from this the current flowing around the circuit is given as:

$$I = \frac{V}{R} = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286\ amps$$

[redacted]

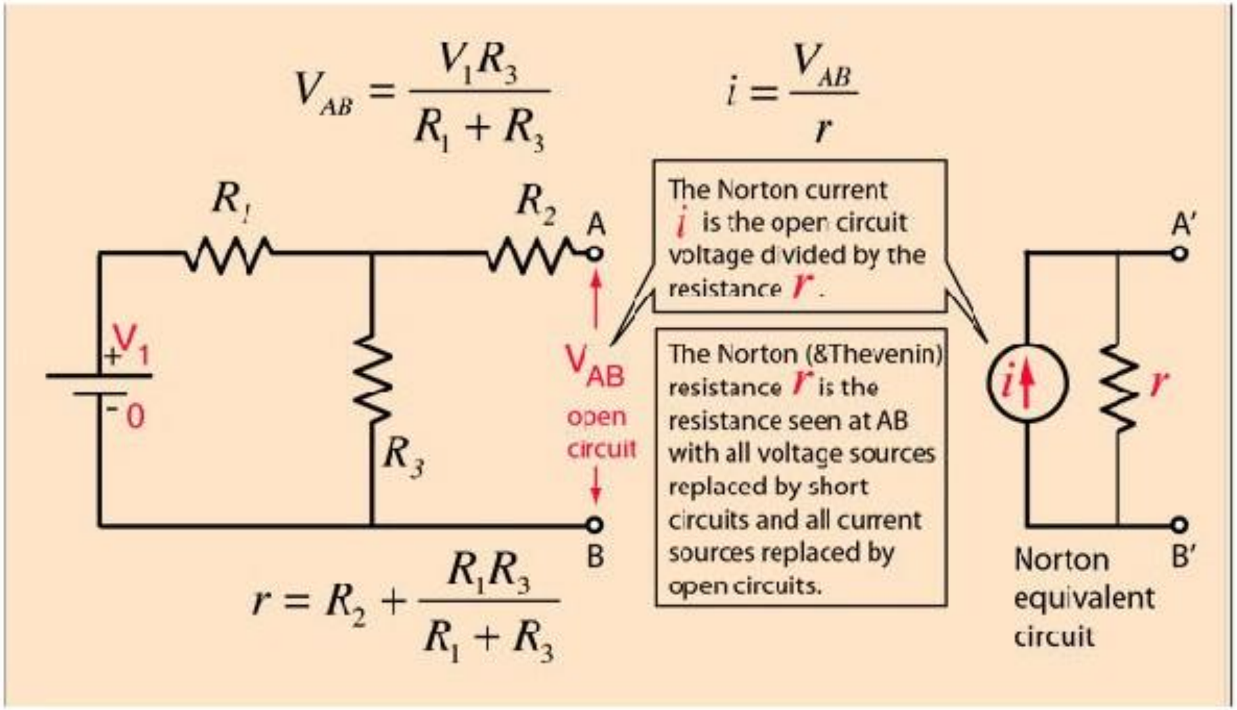
Norton's Theorem states that "*Any linear circuit containing several energy sources and resistances can be replaced by a single Constant Current generator in parallel with a Single Resistor*".

As far as the load resistance, RL is concerned this single resistance, RS is the value of the resistance looking back into the network with all the current sources open circuited and IS is the short circuit current at the output terminals as shown below.



NORTON'S CURRENT

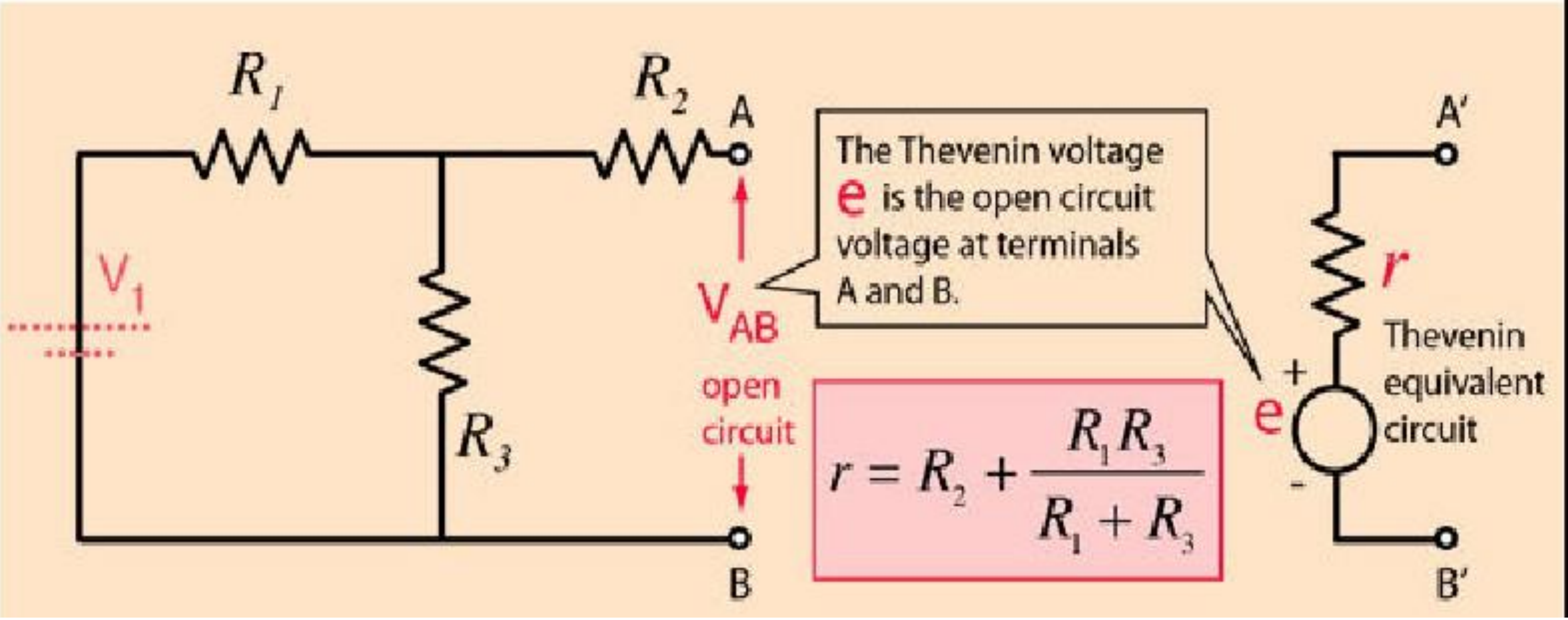
It is the current supplied by the source which will pass through two selected terminals , when they are short circuited.



NORTON'S RESISTANCE

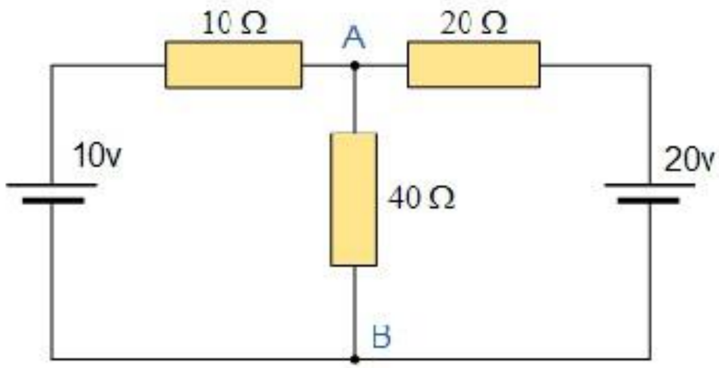
It is generally known equivalent resistance of the network as viewed from two selected terminals, when all the emf sources are replaced by their internal resistance , and current sources by open circuit .

$$r = R_2 + R_1 \| R_3 = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$



EXAMPLE OF NORTON'S THEOREM

To find the Norton's equivalent of the above circuit we firstly have to remove the centre 40Ω load resistor and short out the terminals A and B to give us the following circuit.

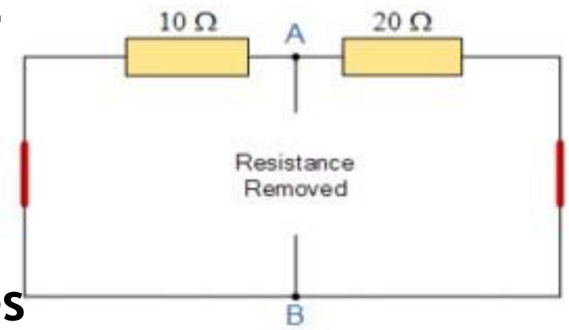


When the terminals A and B are shorted together the two resistors are connected in parallel across their two respective voltage sources and the currents flowing through each resistor as well as the total short circuit current can now be calculated as:

$$I_1 = \frac{10\text{v}}{10\Omega} = 1\text{amp}, \quad I_2 = \frac{20\text{v}}{20\Omega} = 1\text{amp}$$

$$\text{therefore,} \quad I_{\text{short-circuit}} = I_1 - I_2 = 2\text{amps}$$

If we short-out the two voltage sources and open circuit terminals A and B, the two resistors are now effectively connected together in parallel. The value of the internal resistor Rs is found by calculating the total resistance at the terminals A and B giving us the following circuit.



Find the Equivalent Res

10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

Again, the two resistors are connected in parallel across the terminals A and B which gives us a total resistance of:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6.67 \times 40}{6.67 + 40} - 5.72\Omega$$

The voltage across the terminals A and B with the load resistor connected is given as:

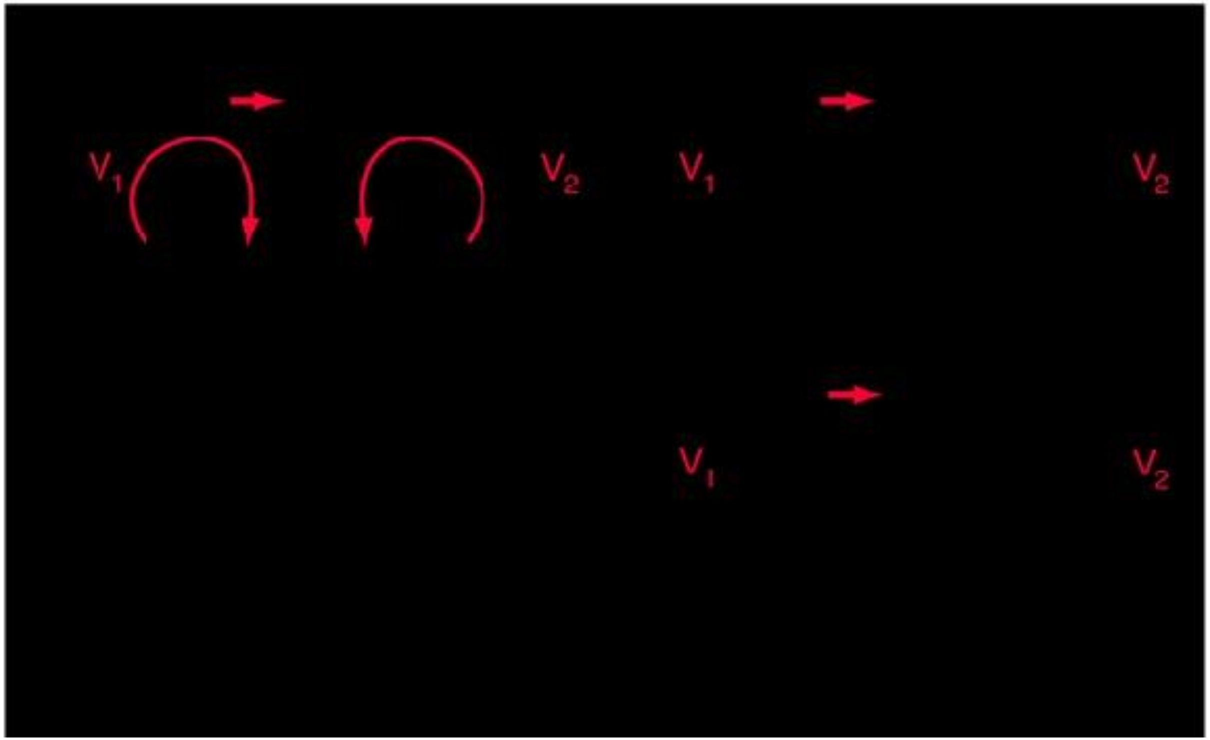
$$V_{A\text{-}B} - I \times R - 2 \times 5.72 - 11.44\text{v}$$

Hence the current flowing in the 40Ω load resistor can be found as:

$$I - \frac{V}{R} = \frac{11.44}{40} = 0.286 \text{ amps}$$

SUPERPOSITION THEOREM

This theorem states that if many voltage sources or current sources are acting simultaneously in a linear network, then resultant current in any branch is the algebraic sum of currents that would flow through it , when each source acting alone, replacing all other emf sources by their internal resistances and keeping the terminals open, from where current source has been removed.



PROCEDURE

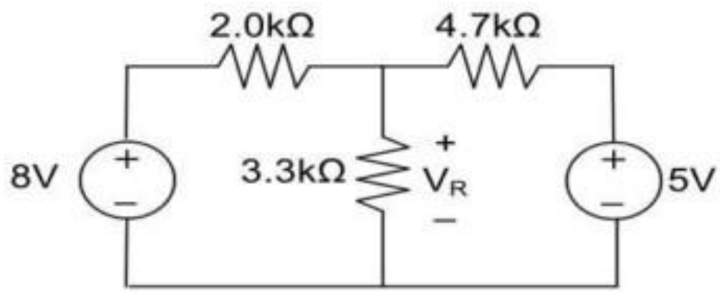
1. Mark the direction of currents in various sections of the network
2. Now consider only one source of emf say E1 , Redraw the circuit and determine the current in those section as I1, I2, I3 respectively
3. Now consider the second source E2, and short circuit E1 and redraw the circuit.
4. Again determine the current bI1, I2, I3 . By superposition theroem the actual flow of current in various sections can be determined by superimposing the current, i.e.,

   I1 = I1'-I1"

   I2 = I2'-I2"

   I3 = I3'+I3"

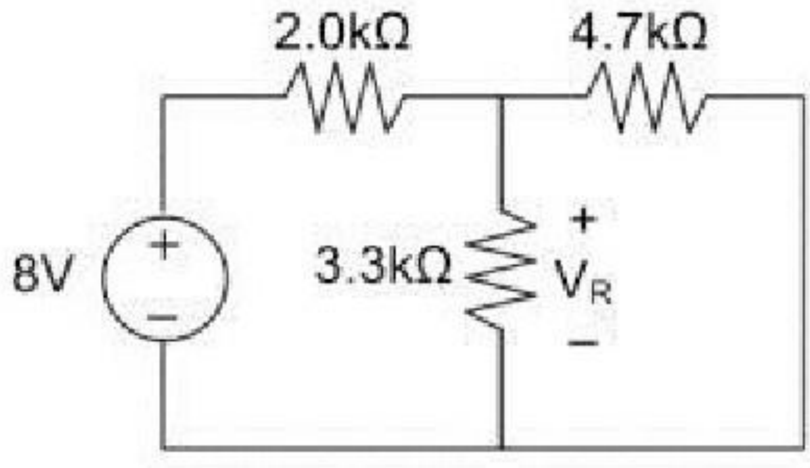
EXAMPLE OF SUPERPOSITION THEOREM



Step 1: Remove the 8V power supply from the original circuit, such that the new circuit becomes as the following and then measure voltage across resistor.

- Here 3.3K and 2K are in parallel, therefore resultant resistance will be 1.245K.
- Using voltage divider rule voltage across 1.245K will be
- V1= [1.245/(1.245+4.7)]*5 = 1.047V

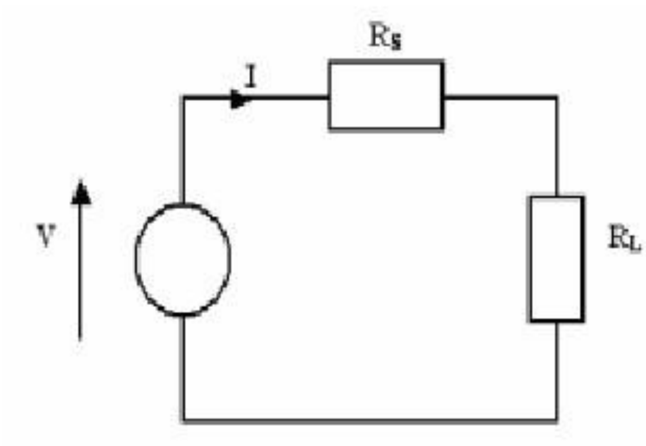
Step 2: Remove the 5V power supply from the original circuit such that the new circuit becomes as the following and then measure voltage across resistor.



- Here 3.3K and 4.7K are in parallel, therefore resultant resistance will be 1.938K.
- Using voltage divider rule voltage across 1.938K will be
- V2= [1.938/(1.938+2)]*8 = 3.9377V
- Therefore voltage drop across 3.3K resistor is V1+V2 = 1.047+3.9377=4.9847

MAXIMUM POWER TRANSFER THEOREM

It deals with transfer of maximum power from a source to load. This theorem in dc circuit states that the load resistance should be equal to the internal resistance of the source for maximum power transfer from source to load . This condition is also referred as resistance matching and it is very important in electronics and communication circuits or obtaining maximum output.



Considering figure B the value of current will be calculated by the equation s

$$I = \frac{V_{TH}}{R_{TH} + R_L} \ldots\ldots\ldots\ldots (1)$$

While the power delivered to the resistive load is given by the equation

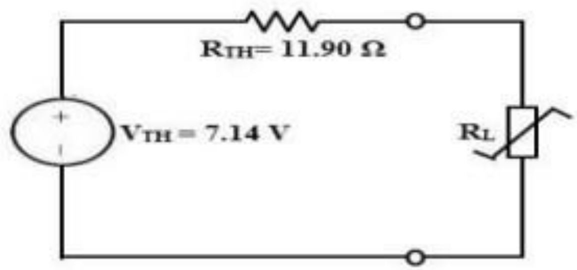
$$P_L = I^2 R_L \ldots\ldots\ldots\ldots (2)$$

Putting the value of I from the equation (1) in the equation (2) we will get

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 x\, R_L$$

EXAMPLE OF MAXIMUM POWER TRANSFER THEOREM

The Thevenin's equivalent circuit with above calculated values by reconnecting the load resistance is shown below.



From the maximum power transfer theorem, RL value must equal to the RTH to deliver the maximum power to the load.

Therefore, $R_L = R_{TH} = 11.90$ Ohms

And the maximum power transferred under this condition is,

$P_{max} = V^2_{TH} / 4 R_{TH}$

$= (7.14)^2 / (4 \times 11.90)$

$= 50.97 / 47.6$

$= 1.07$ Watts